



# MEASURING THE MASS OF JUPITER

**IMPORTANT NOTE:** bring a 3.5” floppy disk with you to class, so you can save your data.

This lab is an indoors lab that will be performed in the MCS (Math) building, room 8, which is a computer lab located in the basement. If you want to practice or repeat this later at home, you can download this free software and other supporting documentation at the web site:

<http://www.gettysburg.edu/academics/physics/clea/juplab.html>

*Purpose:* In this lab, you will determine the mass of Jupiter using Kepler’s 3<sup>rd</sup> law (as revised by Isaac Newton). Essentially, we will use a computer program to follow up on (or, if weather was bad, preview) observations of Jupiter and its moons, by virtually observing Jupiter many times over a period of a few weeks – obviously something that would be impractical for this class.

## **Introductory background material**

This lab will be divided into four parts: learning the software, estimating the data you expect to obtain, obtaining the actual data, and computing results of the data, along with associated errors.

As we did (or will hopefully) observe, Jupiter is a rather large planet – the largest in the Solar System – with several moons. We observed 3 or 4 of its largest moons with our telescopes: Io, Europa, Ganymede, and Callisto and determined how far they were from Jupiter (in arcseconds – which we could have easily converted into “Jupiter diameters – which will be the units we use in this lab). The order listed above for the moons represents the moons from closest circular orbit to furthest circular orbit around Jupiter. [I Eat Green Carrots.] However, they don’t always LOOK like they’re in this order!

Galileo was the first person to observe those moons in the late 17<sup>th</sup> century. After many careful observations, he realized that the moons appeared to move east and west of Jupiter and he correctly interpreted this sideways motion as a one-dimensional, edge-on view of a two-dimensional circular orbit. Therefore, Galileo was the first person to recognize that not everything in the solar system was moving in circles around the Earth. His data, in conjunction with Kepler’s ideas about planetary orbits played a major role in the development of modern science, and these two ideas played a fundamental role in the Copernican Revolution, which has led people to believe that the Earth is not located at a special place in the universe.

Kepler’s third law of planetary motion, as revised by Isaac Newton (in his law of gravity), says that if a smaller object circularly orbits a much more massive object, the orbit’s size and time are related to the mass of the much more massive object in the following way:

$$(\text{Radius of orbit})^3 / (\text{Time for one orbit})^2 = (\text{mass of large object})$$

**Note:** For this equation to be valid as written, you must measure the radius in terms of astronomical units, and you must measure the time in years. The output mass of the large object will be the mass of the object divided by the mass of the Sun. (If you use more standard units, such as meters, seconds, and kilograms, you must multiply a constant to one side of the equation.) Also note that this law is a direct consequence of the law of gravity, because gravity is what keeps objects in orbit around other objects.

Since the mass of the larger object can be determined by carefully observing any object orbiting around it, people were able to calculate the mass of Jupiter (divided by the Sun’s mass). We will replicate this experiment by watching the four Galilean moons and will obtain four values of Jupiter’s mass and will therefore figure out our best guess as to the true value.

**Procedure****Learning the software**

The first thing you need to do is to familiarize yourself with the software. To load the Jupiter simulation software, go to the Start menu → Programs → Natural Sciences → Astronomy → CLEA exercises → Jupiter Moons. You will need to log in (under the file menu). Use your first initial and last name as your login. (When you save data, the computer saves it as your login name.) For instance, my login would be dvakil.

The first thing we will do is to see how a side-by-side motion is related to circular motion, as Galileo determined. We will start by watching a time-lapse animation of the moons. Select file → Preferences → Timing. Make sure the observation interval is 12 hours and the animation timestep is 0.5 hours. To make it easier to identify one moon from the other, also select the moon ID colors from the preferences menu. This will color-code the moons while we use the software. Lastly, turn on the animation in the preferences. Leave the “top view” off for now.

To watch the time-lapse animation, go to File → Run, and set up the software to start on the current date and time. Universal time is Pacific Daylight [i.e. summer-ish] Time plus 7 hours, and Pacific Standard Time plus 8 hours. (When it is noon here, it is 8pm, or 20:00 UT.) When you enter the date and time, a simulation of the Jovian system will appear, magnified 100x (which is approximately what it would look like through the 17mm eyepiece – which has 118x). You can switch to higher magnifications if you wish.

You should see the date and UT, as well as the “Julian day” which is a numerical way to keep track of the date. For the purposes of this lab, you will only need to concern yourself with the last three integers in this Julian date. (Example: on March 31, 2003, the Julian date is 2452729 – you only need worry about the ending 729 part.)

To start/stop the animation, click the “cont” button. A red light above it will turn on when the animation is running. Notice that the moons go both in front of and behind Jupiter – something that is very difficult to see in an actual telescope (especially if the moon’s shadow isn’t visible on Jupiter’s disk). Note that while the animation is running, you will be unable to perform most program tasks. So turn it off when you’re done with it.

Once you’ve become familiar with the left-right motion, turn on the “top view” in the preferences menu. While Earth usually has an edge-on view of the Jovian system, the computer program can show you a view of the system from above Jupiter, giving a different perspective. With both views on, you should be able to see how the circular motion correlates with the east/west positions of the moons.

**Estimating data**

Before we proceed with data taking, you should estimate what the orbits are of each moon. Fill in the top table at the end of this lab. You will need to estimate the time it takes each moon to orbit Jupiter (time period), the radius of the orbit (in Jupiter diameter units), and the Julian date when each moon crosses in FRONT of the center of Jupiter (which is called “T-zero” in this lab – the time the orbit officially begins). Pick your T-zero on a date near today’s date.

To estimate the radius of the orbit, stop the animation and click the mouse on the position where you think the edge of the orbit is. You should see something like “X = 9.90E [Jup Diam.]” This means you clicked 9.9 Jupiter diameters to the east of Jupiter’s center. If there is a moon where you clicked, the moon’s name should appear also. You can ignore the “X = ... , Y = ...” stuff.

To estimate the time period, note the Julian date when the moon starts at one easily recognizable place in its orbit (e.g. the eastern edge, the western edge, or in front and center of Jupiter), and let the animation run

(perhaps more slowly, by picking small animation timesteps) until the moon reaches that same part of the orbit. Compute the amount of time elapsed during the orbit by subtracting the Julian dates.

For your reference, also write down the color of each moon.

### Gathering data

Now that you've estimated what the answers are, we will attempt to measure the actual values. To do this, we will need to deactivate the animation. Run the program again, setting the date and time to the current time. Make sure that the observation time intervals are still 12 hours in the preferences.

Now, the program shows you what the Jovian system looks like at a particular time. What you need to do is record the position of each moon TWICE by following the following procedure:

- 1) Select the highest magnification possible for the target moon
- 2) Click on the moon. Make sure the name of the moon appears
- 3) Write down the moon's location [e.g. 1.38W – record the number of Jupiter diameters], date, time, and Julian Date (last 3 digits + 1 decimal place only). If a moon is not visible, leave it blank. (Make sure you check all of the magnifications!)
- 4) Repeat for each moon on the given date.
- 5) **Before hitting “next”** to proceed with the next observation, taken 12 hours later, **select “Record measurements”** and type in what you wrote down.
- 6) Repeat this process until you have 25 clear night observations. Cloudy nights do not count. Leave those entries blank and/or denote them as cloudy.
- 7) Save your data file periodically, in case something bad happens. (File → Data → Save)

What you have done is record the position of the moon for various times. We will now measure the radius and time period of each moon's orbit based on your data.

### Measuring the actual radius and time of the orbits

Once you have collected and typed in all of your data, make sure your data is saved, and then proceed to the analysis. (File → Data → Analyze) Select Callisto as the first moon, since that will be easiest. Your data should appear on a graph. The vertical axis on the graph measures the moon's position relative to Jupiter. (The zero line – the line in the middle – is where the center of Jupiter is. All measurements are in units of Jupiter diameters.) The horizontal axis measures time, specifically the last 3 digits of the Julian date.

The overall shape of the graph should look like what you saw during the animation – the moon should go east (below the axis), through Jupiter (zero line), to west (above the axis). Since the moon is orbiting in a circle, but we only see it edge on, we are seeing the edge-on projection of a circle, which is a sine curve.

You will now attempt to figure out the real values of the data you estimated earlier. To do so, you must fit the “best” sine curve you can to the data. The computer will help you with this. Start by going to Plot → Fit Sine Curve → Set initial parameters. Enter your estimates. For the amplitude of the sine curve, put in the orbital radius estimate.

You should now see a blue curve superimposed on your data. If your estimates are good, the curve should go through (or near) most of the data points. If your estimates are poor...

Your job is to make the curve look as good as possible by having it go through the data as best you can. You can adjust the “estimates” using the scroll bars at the bottom. Remember, “T-zero” is where the moon crosses in front of Jupiter, “amplitude” is the height of the curve, and “period” is horizontal time between peaks.

(If you made an obvious typo or measure-o, based on what you see on the graph, ask the instructor what to do.)

Adjust the curve until you are satisfied with the results. If you're having trouble, ask your instructor for tips. Record the following information in the second table on page 8: the "best" amplitude, period, and T-zero, as well as the RMS residuals. For the RMS record the data similar to the example given below.

*Example calculation:*

If the RMS reads 6.13275E+02, record 6.13E+02. Extra credit will be awarded for small values of the RMS residual. Penalties will be assessed for abnormally large RMS (E+00 or higher).

RMS residual measures how close your data resemble a perfect sine curve. The smaller the number, the more perfect the sign curve for your data. The numbers are written in scientific notation. For example:

2.45E-01 means  $2.45 \times 10^{-1} = 0.245$

8.23E-02 means  $8.23 \times 10^{-2} = 0.0823$

9.56E-03 means  $9.56 \times 10^{-3} = 0.00956$

In practice, you will only get E-01, E-02, or maybe E-03. The number after the "E" should be as negative as possible (i.e. E-03). Extra credit will be awarded as follows: Any RMS less than 0.03 receives 2 points. Anything less than 0.06 receives 1 point. Anything less than 0.1 receives ½ point. Penalties will be assessed for abnormally large RMS (E+00 or higher).

When you have finished entering your best results in the table, **print out your graph**. Complete the table for all four moons.

To calculate the mass of Jupiter, you will need to convert units so that they fit the version of Kepler's 3<sup>rd</sup> law that we are using – see Data Sheet B. The answer you get will be the mass of Jupiter in solar units (i.e. Jupiter's mass divided by the mass of the Sun). In other words, if your answer is  $M = 2.0$ , your calculations say Jupiter is twice as massive as the Sun. If you get 0.005, then Jupiter is 1/200<sup>th</sup> of the mass of the Sun.

*Questions:* (for the questions, use your average value of Jupiter's mass, unless indicated otherwise)

1. [1 pt] Based on your data, how many times more massive than Jupiter is the Sun?
2. [1 pt] Based on your data, how many times more massive than the Earth is Jupiter? The Earth has a mass of  $3.00 \times 10^{-6}$  Suns. [These 2 questions each involve only one simple calculation.]
3. [1 pt] What are the actual answers to questions 1 and 2? [Research!]
4. [1 pt] A) Based on your common sense, if Jupiter got heavier, but the moons didn't change location, would their orbital times increase or decrease? B) Justify your choice. Do NOT refer to Kepler's laws.
5. [1 pt] A) Based on your common sense, if Jupiter got heavier, but the moons kept the same orbital times, would they have to be closer or further away? B) Justify your choice.
6. [1 pt] What does the equation for Kepler's 3<sup>rd</sup> law say about questions 4 and 5?
7. [1.5 pts] A) How consistent were your 4 values for Jupiter's mass? In other words, did you get four very similar answers for the four moons, or were the data spread out? B) Compute the percent difference between the largest and smallest value of Jupiter's mass. (Hopefully it's a few percent, not much more.) C) Why might your answers have been consistent? Why might they be inconsistent?

8. [1 pt] There are moons around Jupiter beyond the orbit of Callisto. A) Will they have larger or smaller orbit times than Callisto? B) Why?
9. [1.5 pts] If a brand new moon is found at a distance of 3.60 Jupiter diameters away from Jupiter's center, A) how long would it take this moon to orbit Jupiter? B) Answer this question if we have observed Jupiter's moons: would you be able to detect its motion during a 3-hour observing period (e.g. during our class)?
10. [1 pt] What is the ratio of Europa's orbital period to Io's orbital period? If you got the right answers, this ratio should be a "nice" number that plays a large role in the volcanic activity on Io and, along with the Ganymede/Europa ratio, is also related to why the interior of Europa is warm enough to melt ice into water. (Io is the most volcanic body in the Solar System. Europa is probably the best place in the Solar System to look for life, outside of Earth because there is warm water there, under the surface ice.)
11. [1 pt] The data points from Io's amplitude/time graph (which you printed, hopefully) do not resemble a smooth curve like the data from the other 3 moons did. Io's data should be much more scattered. Why? [This is related to a phenomenon called "Nyquist sampling" in digital signal processing.]
12. [Optional] Feel free to provide feedback about this lab or this write-up. Thanks!



**Data Sheet A**

[6.5 pts] Collect 25 clear nights of data. Record nights that were cloudy as “Cloudy.” Don’t forget to record if the planet is “E”ast (negative/below axis) or “W”est (positive/above axis) of Jupiter.

<b>Data Number</b>	<b>Date</b>	<b>Time</b>	<b>Julian Date (XXX.X)</b>	<b>Io</b>	<b>Europa</b>	<b>Ganymede</b>	<b>Callisto</b>
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
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27							
28							

**Data Sheet B**

[2 pts] Estimates from graph or animation or small time intervals.

Moon	Moon color	Time period (days) [T]	Radius (JD= Jupiter Diameters)	T-zero (date = XXX.X)
IV Callisto				
III Ganymede				
II Europa				
I Io				

Note: for the “Radius” column, you can ignore the “E” and “W”. But not for Data Sheet A!

[2 pts, plus extra credit for low RMS values] Actual calculations after fitting sine curve

Moon	T (days)	Radius <sub>JD</sub>	T-Zero (date)	RMS residual (X.XXE-XX)
IV Callisto				
III Ganymede				
II Europa				
I Io				

[1 pt] Convert units using: 365 days = 1 year; 1046 Jup Diam = 1 AU. In the table below, make sure there are at least 3 non-zero digits. (In other words, don’t round to 0.054 – write 0.0536. But 0.506 is ok – you don’t have to write 0.5062.) Sample calculation: if T was 7 days, then this is  $7/365 = 0.0192$  years. If you got 25 JD’s, that’s  $25/1046 = 0.0239$  AU.

Moon	T (years)	Radius (AU)
IV Callisto		
III Ganymede		
II Europa		
I Io		

Use Kepler’s 3<sup>rd</sup> law:  $Mass = radius^3/period^2$

Mass of Jupiter (in Solar units): SHOW ALL WORK! [1 pt total for 4 moons, ½ pt for average]

From Callisto:

From Ganymede:

From Europa:

From Io:

Average of all 4 moons:

(Reminder: If Jupiter were half as massive as the Sun, you’d get 0.5 here. If Jupiter were 28% of the Sun’s mass, you’d get 0.28. Use your common sense to see if your answer is reasonable!)

**Don’t forget to answer the questions at the end of the lab and attach your four graph printouts.**