We now ask how many arrangements (permutations) there are of 3 and 4 persons sitting about a circular table? Below we see the 2 cases; if \( n = 2 \), we see 2 arrangements and if \( n = 3 \), there are 6 arrangements.

We know that there are \( n! \) different ways to arrange \( n \) people in a line, since each arrangement corresponds to a permutation of the \( n \) people. But how many ways are there to arrange \( n \) people in a circle?

The answer is not \( n! \). The reason is that 2 circular arrangements are considered to be the same if one can be obtained from the other by a rotation.

Problem. At a meeting of diplomats, the 6 members are to be seated around a circular table. Since the table has no ends to confer particular status, it doesn’t matter who sits in which chair. But it does matter how the diplomats are seated relative to each other. In other words, two seatings are considered the same if one is a rotation of the other. How many different ways can the diplomats be seated?

Solution. Call the diplomats A, B, C, D, E and F. Since only relative position matters, you can start with any diplomat (say A) and place that diplomat anywhere (say in the top seat of the table-north) and then consider all arrangements of the other diplomats around that one. B through F can be arranged in the seats around diplomat A in all possible orders. So there are \( 5! = 120 \) ways to seat the group.

Circular arrangements such as this are known as circular permutations, and the previous example shows that there are \( (n-1)! \) circular permutations of \( n \) objects.

Problem. In how many ways can six women, six men and a dog sit around a table such that no two women sit next to each other?

Solution. There are many different ways to seat the group. Just treat the dog as a man. Then seat the men first (there are \( \frac{7!}{1!} \) ways) and then seat six women in the seven space between men (and dog). There are \( P(7, 6) = 7! \) ways. Another way to do this is to seat the dog first, there is only one way to do so. Then sit the six men—there are \( 6! \) ways to do so (it becomes a linear permutation after the dog has been seated). Then seat the six women in the 7 spaces in the between of men and/or dog. There are \( P(7, 6) = 7! \) ways to do so. Thus there are \( 6! \cdot 7! = 3628800 \) ways to seat them.

Seating the 6 men first (or 6 women first), 6 women (or 6 men) next in the 6 spaces in the between (there are \( 5! \cdot 6! \) ways to do so) and then seating the dog anywhere in the 12 spaces in between will not count the situation that the dog sits between 2 women and thus will not give the correct counting.