Circles $A, B, C$ are externally tangent to each other and internally tangent to circle $D$. Circles $B, C$ are congruent. Circle $A$ has radius 1 and passes through center of circle $D$.

Find radius of circle $B$.

$EF = 1$ is given.

Radius of Circle $B$ is $\frac{8}{9}$.

Let $E, H, F$ be the centers of circles $A, B, D$, respectively.
Let $G$ be point of tangency of circles $B$ and $C$.
Let $FG = x$ and $GH = y = HS$, both radii of circle $B$.

Since $EF = 1$ and $F$ is the center $\rightarrow$ radius of $D$, big circle, is 2.
So $FK = 2$, so since $HK = y$, we know $FH = 2 - y$.

**Pythagorean Theorem**

$AEGH \rightarrow y^2 + (x + 1)^2 = (y + 1)^2$

$y^2 + x^2 + 2x + 1 = y^2 + 2y + 1$

$x^2 + 2x = 2y^2 \rightarrow \text{both } = 2y$

$x^2 + 2x = \frac{2y}{2} \rightarrow \text{both } = 2y$

$\Delta GFH \rightarrow x^2 + y^2 = (2 - y)^2$

$x^2 + y^2 = 4 - 4y + y^2$

$4y = 4 - x^2$

$2y = 2 - \frac{1}{2}x^2$

$x^2 + 2x = 2 - \frac{1}{2}x^2$

$2x^2 + 4x = 4 - x^2$

$3x^2 + 4x - 4 = 0$

$3x - 2)(x + 2) = 0$

$x = \frac{2}{3}, x = -2$

$x = \frac{2}{3}$

$4y = 4 - \left(\frac{2}{3}\right)^2$

$y = 4 - \frac{4}{9}$

$y = \frac{8}{9}$.
<table>
<thead>
<tr>
<th>Five Possible Cases</th>
<th>A says &quot;D is murder!&quot;</th>
<th>B says &quot;D is innocent&quot;</th>
<th>C says &quot;It wasn't E&quot;</th>
<th>D says &quot;A is lyin'&quot;</th>
<th>E says &quot;B is telling T&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) A is killer</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>2) B is killer</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>3) C is killer</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>4) D is killer</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>*5) E is killer</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Check each statement's truthfulness for each of the five cases. We need a horizontal row with 2 Fs and 3 Ts.

(E must be the killer)