Chap 4.1 Solving Linear Inequality (>, <, ≥, ≤)

The steps to solve inequality are just like those we use to solve an equation:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3x - 5 = 16)</td>
<td>(3x - 5 &gt; 16)</td>
</tr>
<tr>
<td>(3x = 16 + 5)</td>
<td>(3x &gt; 16 + 5)</td>
</tr>
<tr>
<td>(3x = 21)</td>
<td>(3x &gt; 21)</td>
</tr>
<tr>
<td>(x = 7)</td>
<td>(\frac{3x}{3} &gt; \frac{21}{3})</td>
</tr>
<tr>
<td></td>
<td>(x &gt; 7)</td>
</tr>
</tbody>
</table>

In addition to getting an Inequality Answer, we like to indicate/show the set of Numbers that the answer refers to on the Number Line with a Number Line graph:

\[ \infty \to -5 \to -2 \to 0 \to 5 \to 6 \to 7 \to 8 \to \infty \]

Or use a paren with Infinity:

\[ \infty \to -5 \to -2 \to 0 \to 5 \to 6 \to 7 \to 8 \to \infty \]

The \( \leq \) or \( \geq \) at 7 indicates we are using \( > \) if it had been \( \geq \) symbol the we would use \( \circ \) or [ at 7 on the line!

\( X \geq 7 \)

Checking your answer: Choose a number from the area of the number line in your solution (but not the endpt!)

Let \( x = 8 \)

\( 3(8) - 5 > 16 \)
\( 24 - 5 > 16 \)
\( 19 > 16 \) True
**Special Rule for All Inequality Problems**

If, while solving an inequality, you need to multiply or divide both sides by a "negative" number, then you must **reverse the direction of the inequality symbol**.

**Example**

\[
-5x + 3 \leq 5
\]

\[
-5x \leq 5 - 3 \quad \text{move from to other side}
\]

\[
-5x \leq 2 \quad \text{add like terms}
\]

\[
-5 \quad \text{divide both sides by -5}
\]

\[
x \geq -\frac{2}{5} \quad \text{reverse direction}
\]

Check: let \( x = 0 \)

\[
-5(0) + 3 \leq 5 \quad 3 \leq 5 \quad \text{True}
\]

Next, we introduce new notation which eliminates the need to draw line graphs. It is called **Interval Notation**.

Instead of using the line graph for \( x > 7 \)

\[
\infty \quad 0 \quad 7 \quad 8 \quad \infty
\]

we use \( (7, \infty) \) **Interval Notation**

INDICATES

\[\uparrow\]

left end pt. is open

Another example

\( x \leq -2 \)

Like Graph

\[
\infty \quad -3 \quad -2 \quad -1 \quad 0
\]

we use \( (-\infty, -2] \) **Square Bracket**

\[\downarrow\]

indicates you can be \( = -2 \)

\[\uparrow\]

left end pt. is open

\[\uparrow\]

right end pt.
Now, let's do a more complex inequality problem:

Example:

\[ \frac{1}{3}x + 1 < 4 + 5x \]

Because there is a fraction in a term, multiply all terms by 3

\[ \frac{1}{3} \quad \uparrow \quad \uparrow \quad \uparrow \quad (3) \]

\[ x + 3 < 12 + 15x \]

Jump terms across/putting all variable terms on the left side

\[ x - 15x < 12 - 3 \]

Combine like terms

\[ -14x < 9 \]

Divide by -14 on both sides

\[ -14x < \frac{9}{-14} \]

\[ x > \frac{-9}{-14} \Rightarrow \left( \frac{-9}{14}, \infty \right) \]

Points to right end of number line

Note that if you always put your variables on the left side, then the direction of the inequality symbol will be pointing at the infinity that must be included in the answer.

In class work #1

Pg 332 #60) Solve

\[ \frac{1}{3}x + 1 < 4 + 5x \]
4.2 Solving Compound Inequalities

Compound statements are those statements that join two other simple statements with the words "AND" or "OR".

The logic of "AND"

Given the compound statement:

"I WILL MOW THE LAWN" AND "I WILL WASH THE CAR"

There are four possible outcomes:

1. I mow the lawn AND I wash the car
2. I do not mow the lawn AND I wash the car
3. I mow the lawn AND I do not wash the car
4. I do not mow the lawn AND I do not wash the car

Note the compound statement S1 AND S2 is only true when both S1 and S2 are true!

Putting this logic into the world of inequalities, we can solve

\[ x > 3 \text{ AND } x < 7 \]

by choosing only x values that will make \( x > 3 \) true and make \( x < 7 \) true at the same time!

\[ x > 3 \]

(3, \infty)

\[ x < 7 \]

(-\infty, 7)

\[ (3, 7) \] is the answer for the compound statement!

\( x \) > 3 \text{ AND } x < 7

\[
\begin{array}{|c|c|c|}
\hline
S1 & S2 & S1 AND S2 \\
\hline
T & T & T \\
F & T & F \\
T & F & F \\
F & F & F \\
\hline
\end{array}
\]
THE LOGIC OF "OR"

Given the Compound Statement

"I will mow the lawn OR I will wash the car"

THERE ARE THE SAME FOUR POSSIBLE OUTCOMES

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
<th>S1 OR S2</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Note that the Compound Statement
S1 OR S2 is True if either
or both statements S1 and S2
are true. It is only false
when both statements are false.

Putting this logic into the world of INEQUALITIES, we can

SOLVE \( x > 5 \) OR \( x < -2 \)

By choosing \( x \) values that
make either or both statements true.

\[ x > 5 \]
\[ \infty \quad -2 \quad -1 \quad 0 \quad 4 \quad 5 \quad 6 \]

\[ x < -2 \]
\[ -\infty \quad -2 \quad -1 \quad 0 \quad 4 \quad 5 \quad 6 \quad \infty \]

which gives us the
solutions set
\[ (-\infty, -2) \cup (5, \infty) \]

Example Problems:
\( 5(x+1) \leq 2(x+1) \) AND \( x \geq 2x - 5 \)
\( 5x+5 \leq 2x+9 \)
\( x-2x \geq -5 \)
\( 3x \leq -1 \)
\( x \leq -\frac{1}{3} \)
\( x \leq \frac{5}{3} \)

\( 4x < -12 \) OR \( \frac{x}{2} > 4 \)
\( 4x < -12 \)
\( x < -3 \)

\( x < -3 \) OR \( x > 8 \)

\[ (-\infty, -3) \cup (8, \infty) \]
IN CLASS WORK #2

PG 344 #29 Solve
6x + 1 < 5x - 3 AND $\frac{x}{2} + 9 \leq 6$

IN CLASS WORK #3

PG 344 Solve
#42
3x + 4 < -2 or 3x + 4 > 10