5.4) Multiplying Polynomials:

\[ 23(18) = 23(20 - 2) = 460 - 46 = 414. \]

Distributive law:

\[ a(b + c) = ab + ac. \]
\[ a(b - c) = ab - ac. \]
\[ a(b + c + d) = ab + ac + ad. \]

Examples:

\[ 3x^2(-2x^2 - 4x + 1) \]

1 term \times 3 terms = 3 terms.

\[ = (3x^2)(-2x^2) + (3x^2)(-4x) + (3x^2)(1). \]
\[ = -6x^4 - 12x^3 + 3x^2 \]

Example: 2 terms on outside.

\[ (3x^2 - 4)(7x^3 + 31x) \]

2 terms \times 2 terms = 4 terms.

left Each term on the left takes time to distribute into terms on the right.

\[ (3x^2 - 4)(7x^3 + 31x) \]

\[ = 3x^2(7x^3) - 3x^2(31x) - 4(7x^3) - 4(31x) \]
\[ = 21x^5 + 93x^3 - 28x^3 - 124x \]

\[ \text{like terms:} \]
\[ = 21x^5 + 65x^3 - 124x \]

F \quad F \quad O \quad I \quad L \quad L \quad O\]
Example:

\[
(3x^2 - 5)(-2x^2 - 4x + 1)
\]

2 terms \times 3 terms = 6 terms.

\[
= (3x^2)(-2x^2) \quad (3x^2)(-4x) \quad (3x^2)(+1) \quad (-5)(-2x^2) \quad (-5)(-4x) \quad (-5)(+1)
\]

\[
= -6x^4 - 12x^3 + 3x^2 + 10x^2 + 20x - 5
\]

like terms

\[
= -6x^4 - 12x^3 + 13x^2 + 20x - 5
\]

Special Cases:

18 \cdot 22

\[
(20 - 2)(20 + 2) = 20^2 - 2^2 \text{ because }
\]

\[
= 20^2 + 40 - 40 - 2^2 = 400 - 4 = 396.
\]

\[
(a - b)(a + b) = a^2 - b^2 \text{ (The product of the sum and difference of two terms).}
\]

Another Example:

\[
(53)(47) = (50 + 3)(50 - 3)
\]

Try this 1:

\[
41 \times 39 = (40 + 1)(40 - 1) = 40^2 - 1^2
\]

\[
= 1600 - 1 = 1599.
\]

Try this 2:

\[
49 + 49
\]

\[
(50 - 1)(50 - 1) = (50 - 1)^2 \text{ Square of a difference}
\]

\[
(a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2
\]

\[
= (50)^2 - 2(50)(1) + (1)^2 = 2500 - 100 + 1
\]

\[
= 2401.
\]
\((53)^2 = (50+3)(50+3)\). The square of a sum.
\((x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2\)
\((50+3)(50+3) = (50)^2 + 2(50)(3) + (3)^2\)
\[= 2500 + 300 + 9 = 2809\]

\((x+y)^2 = x^2 + 2(x)(y) + y^2\)
\((x+y)^2 = x^2 + 8x + 16\)

In Math 70:

\((3p^2 + 4q^2)^2\) =

long way
\((3p^2 + 4q^2)(3p^2 + 4q^2)\)
\[= 9p^4 + 12p^2q^2 + 12p^2q^2 + 16q^4\]
\[= 9p^4 + 24p^2q^2 + 16q^4\]

Using the special product formula.
\((3p^2 + 4q^2)^2 = 9p^4 + 24p^2q^2 + 16q^4\)

In class problems:

P 445

1. \(\#36\) multiply \((a+2)(3a^2 + 4a - 2)\)

2. \(\#50\) \((6p^2 - 3)^2\)
Section 5.5: Factoring

Factoring the GCF

To factor is to show a multiply operation between two or more two quantities for a given quantity.

Factor:

\[ 24 = 8 \cdot 3 \quad \frac{24x^3}{24} = \frac{8x^2(3x)}{(2x)(12x^2)} \]
\[ = 12 \cdot 2 \quad = (12x^2)(2x), \quad (2x)(12x^2) \]
\[ = 4 \cdot 6 \quad = 6x^2(4x), \quad (4x^2)(6x) \]
\[ = 24 \cdot 1 \quad = (-8x^2)(-3x) \]

You can bring all the negative factors also.

When you have Two Terms.

Factor: product of two \((a \cdot b)\)

\[ 14x^2 - 7x = \]

\[ (7x)(2x) - (1)(7x) = 7x (2x - 1) \quad \text{Using Distributive} \]
\[ \text{common factor} \]

\[ 27z^3 + 12z^2 + 3z = \]
\[ = (3z)(9z^2) + (3z)(4z) + (3z \cdot 1) \]
\[ = 3z (9z^2 + 4z + 1) . \]

Now: Two Terms

\[ \text{pig} \cdot 7x + \text{pig} \cdot 2 = \text{pig} (7x + 2) \]
\[ (x+1) \cdot 7x + (x+1) \cdot 2 \quad = 7x^2 + 7x + 2x + 2 \]
\[ = (x+1)(7x+2), \quad = 7x^2 + 2x + 7x + 2 \]
Factoring by grouping:

\[ 2x - 2y + xy - y^2 \quad (4 \text{ terms}) \]
\[ = 2(x-y) + y(x-y) \]
\[ = (x-y)(2+y) \]

In class problems

3. P. 457 #91) Factor A GCF:
\[ 45 \times 10y^3 - 63x^7y^7 + 81x^{10}y^{10} \]

4. P. 457 #72) Rearrange and factor by grouping:
\[ h^3 - 8 + h - 8h^2 \]