9.1) Algebra of functions:

1. Add and subtract functions.
2. Multiply and divide.
3. Composite functions.

\[ f(x) = 3x - 15 \]
\[ g(x) = -8x + 1 \]

Compute \( f(-2) = 3(-2) - 15 = -6 - 15 = -21 \)
Compute \( g(5) = -8(5) + 1 = -40 + 1 = -39 \).

A) Add two functions:

\[ (f + g)(x) \]

\[ \text{Compute the function of } f(x) + g(x), \]
notation of adding two functions.

\[ (f + g)(x) = f(x) + g(x) \]
\[ = 3x - 15 + (-8x + 1) \]
\[ = -3x - 15 - 8x + 1 \]
\[ = -11x - 14 \]

B) Compute the function \( (f - g)(x) = f(x) - g(x) \)

\[ = 3x - 15 - (-8x + 1) = 3x - 15 + 8x - 1 \]
\[ = 11x - 16 \]

C) Compute the function \( (f \cdot g)(x) \) the product (multiplication).

\[ f(x) \cdot g(x) = (3x - 15)(-8x + 1) \]
\[ = -24x + 120x + 3x - 15 \]
\[ = -24x + 123x - 15 \]
d) Compute fractions \( \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} \)

\[
= \frac{3x - 15}{-8 + 1}
\]

In class Problem:

Problem 8.35 - \( f(x) = 2x + 1 \)
\( g(x) = x - 3 \)

Compute \((g - f)(x)\)

3) Composition:

One function is substituted into another function ("fed" into)

Real life Analogy \(\rightarrow\) Laundry.

\[ g(x) \rightarrow \text{washing machine} \rightarrow \text{clean and dry} \]

\[ f(x) \rightarrow \text{drier} \rightarrow \text{clean and dry} \]

\[ f(x) = 3x - 15 \]
\[ g(x) = -8x + 1 \]

Let \( x = 2 \rightarrow \) The dirty Jersey.
Computing \( g(2) = -8(2) + 1 = -16 + 1 = -15 \)

\( x \) is clean, but \( g(x) \) is wet jersey.
\[ f(x) = f(-15) = 3(-15) - 15 = -45 - 15 = -60. \]

(clean dry jersey).

The notation for that mathematical sequence of events is \((f \circ g)(x)\), we say \(f \circ g \circ f\) of \(x\).

old notation \(f(g(x))\) \(g\) operates on \(x\) first.

since \(g\) is closest to the "\(x\)" , then make \(g\) first.

\((h \circ f \circ g)(x)\) \(g\) first, then \(f\), then \(h\).

\[ f(x) = 3x - 2 \]
\[ g(x) = x^2 + x \]

compute \((f \circ g)(4)\)

1st compute \(g(4)\)

\[ g(4) = 4^2 + 4 = 16 + 4 = 20 \]

\[ f(20) = 3(20) - 2 = 60 - 2 = 58. \]

In class Problem:

\[ f(x) = 2x + 1 \]
\[ g(x) = x^2 - 1 \]

compute \((g \circ f)(-3)\)
New Laundry Machine!

We want to create a "Combo" function → one function that will do the job done by two separate functions working sequentially.

\[(f \circ g)(x)\]  
\[\Rightarrow \text{old notation } f(g(x))\]  
\[g(x) \text{ being fed into } f.\]

\[f(x) = 3x - 15\]  
\[g(x) = -8x + 1\]  
\[f(g) = 3g - 15\]

Do the same thing with letter "g" that you did with "x".

Replace the letter g with the expression for g:

\[f(g) = 3g - 15 = 3(-8x + 1) + 15 = -24x + 3 - 15 = -24x - 12.\]

Use this to proof by Example

\[(f \circ g)(2) = -24(2) - 12 = -48 - 12 = -60.\]

**Example 2:**

\[f(x) = 3x - 2\]
\[g(x) = x^2 + x\]
\[f \circ g(x) = f(g) = 3g - 2\]

\[f \circ g(x) = 3(x^2 + x) - 2\]
\[= 3x^2 + 3x - 2\]

proof by example. \(f \circ g(4)\)

\[f \circ g(4) = 3(4)^2 + 3(4) - 2\]
\[= 3(16) + 12 - 2\]
\[= 48 + 12 - 2 = 60 - 2 = 58.\]
Example3) \( f(x) = 2x + 1 \)
\( g(x) = x^2 - 1 \)

\((gof)(x) = g(f(x))\)
\(g(f) = f^2 - 1\)
\(g(f^2) = (2x + 1)^2 - 1\)

\(g(f^2) = 4x^2 + 1 + 4x - 1\)
\(= 4x^2 + 4x\)

Proof by Comparison:
\((gof)(-3) = 4(-3)^2 + 4(-3) = 36 - 12 = 24\)

Section 9.2)

We said that these functions are inverse functions.
One function reverses the operation of the other.

\(f(x) = 3x - 15\)
\(g(x) = \frac{x + 15}{3}\)

Proof by Example.

\(f(29) = 3(29) - 15 = 87 - 15 = 72\)
\(g(72) = \frac{72 + 15}{3} = \frac{87}{3} = 29\).
Special notation is called $g(x)$ is $f^{-1}(x)$.

Inverse of $f(x)$ (look likes a power but it is not the power).

Also $f(x) = g^{-1}(x)$.

* start with

\[ g(21) = \frac{21 + 15}{3} = \frac{36}{3} = 12 \]

\[ f(12) = 3(12) - 15 = 36 - 15 = 21. \]

\[ f(x) = 3x - 15 \]

1. multiply $x$ with 3 first
2. subtract 15.

\[ g(x) = \frac{x + 15}{3} \]

Exact opposite operation in the opposite direction

1. Add 15
2. divide by 3.

How to find the Inverse function:

1. start by writing your function as $y = \ldots$
\[ y = 3x - 15 \]

2. solve for $x$
\[ 3x = y + 15 \]
\[ x = \frac{y + 15}{3} \]

\[ f^{-1}(x) = \frac{2}{3} \frac{x + 15}{3} \]
Find inverse function for

**Example 2** \( f(x) = \sqrt{5x-2} + 1 \)

\[ y = \sqrt{5x-2} + 1 \]

Isolate \( x \).

\[ y - 1 = \sqrt{5x-2} \]

Square both sides.

\[ (y-1)^2 = (\sqrt{5x-2})^2 \]

\[ y^2 - 2y + 1 = 5x - 2 \]

\[ 5x = y^2 - 2y + 1 + 2 \]

\[ 5x = y^2 - 2y + 3 \quad \text{divide by 5} \]

\[ x = \frac{y^2 - 2y + 3}{5} = f'(y) \quad \text{or} \quad f'^2(x) = \frac{x^2 - 2x + 3}{5} \]

In class problem:

**848 #45**

\[ f(x) = x^3 + 8 \]

\[ f^{-1}(x) \]