10-3) Hyperbolas: (Twin curves - Each Similar). "To Parabola."

Whenever the y term is positive:
\[- \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1\]

Covertices.

These lines are called Asymptotes
An imaginary boundary that limit a function or a curve.

OR

Whenever the x term is negative:
\[\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1\]

Covertices.

\[\pm \frac{(x-h)^2}{a^2} \pm \frac{(y-k)^2}{b^2} = 1\]

Hyperbolas the terms are opposite signs.

Example: \[\frac{(x+3)^2}{4} - \frac{(y-1)^2}{9} = 1\]

Center \((-3, 1)\)

\[a = 2 \quad b = 3\]
Hyperbolas from the Homework sheet:

2. \(-9x^2 - 54x + 4y^2 - 8y - 113 = 0\)

\((-9x^2 - 54x) + (4y^2 - 8y) = 113\)

\(-9(x^2 + 6x) + 4(y^2 - 2y) = 113\)

\(\frac{1}{2}(6) = 3 \rightarrow (3)^2 = 9\)

\(\frac{1}{2}(-2) = -1 \rightarrow (-1)^2 = 1\)

\(-9(x^2 + 6x + 9) + 4(y^2 - 2y + 1) = 113 - 81 + 4\)

\(-9(x + 3)^2 + 4(y - 1)^2 = 36\)

divide by 36

\(-\frac{(x+3)^2}{9} + \frac{(y-1)^2}{9} = 1\)
the center is at point (-3, 1)

$a = 2$, $b = 3$. Draw the imaginary box thru the 4 points like a rectangle. Then draw the imaginary diagonal lines thru the corners of the box (they cross in the center). Then draw the curves from the convergence out toward the lines like this.

Draw because $x$ is negative and $y$ is positive.
Graph Hyperbola

\[ x^2 - 4y^2 + 2x - 8y - 7 = 0 \]
\[ (x^2 + 2x) - 4(y^2 + 2y) = 7 \]
\[ (x^2 + 2x + 1) - 4(y^2 + 2y + 1) = 7 + 1 - 4 \]
\[ (x + 1)^2 - 4(y + 1)^2 = 4 \]
\[ \frac{(x+1)^2}{4} - \frac{(y+1)^2}{1} = 1 \]

Center = \((-1, -1)\)  
Horizontal orientation  
\(a = 2\), \(b = 1\)

In Class Problem #28 pg 971

#28
\[ \frac{(y-2)^2}{4} - \frac{(x+1)^2}{1} = 1 \]
9.3) **Exponential Function:**

\[ y = 2^x, \quad y = 3^x, \quad y = \left(\frac{1}{2}\right)^x \]

- \( x \) is the \( n \)th exponent of the expression.
- \( y = 2^x \) \( 2 \) is the base
- \( x \) is the exponent.
- \( y = 2 \) to the \( x \).

**Function table (\( x, y \) chart):**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( y = 2^0 = 1 )</td>
</tr>
<tr>
<td>1</td>
<td>( y = 2^1 = 2 )</td>
</tr>
<tr>
<td>2</td>
<td>( y = 2^2 = 4 )</td>
</tr>
<tr>
<td>3</td>
<td>( y = 2^3 = 8 )</td>
</tr>
<tr>
<td>-1</td>
<td>( y = 2^{-1} = \frac{1}{2} )</td>
</tr>
<tr>
<td>-2</td>
<td>( y = 2^{-2} = \frac{1}{4} )</td>
</tr>
<tr>
<td>-3</td>
<td>( y = 2^{-3} = \frac{1}{2^3} = \frac{1}{8} )</td>
</tr>
</tbody>
</table>

---

**Six-flag Amusement park**

---

**Superman Thrill Ride**

---

**Asymptotic side**

**Base part**

**Powerful side exp.**
9.3) Applications of exponential Functions:

Money /Savings Accounts/.

Assume you put some money \( P \) into an account that is paying interest rate \( r(\%) \) annually.

They will pay you a fraction of your interest over fixed periods of a year.

Suppose:

Bank pay 12\% annually.
If combined it quarterly (4 times per year \( \Rightarrow \) every 3 months)
then they pay you 3% every 3 months.

\[
\frac{12\%}{4} = 3\% \\
\frac{12\%}{12} = 1\% \\
\text{If paid daily } \frac{12\%}{365} = 0.038\%.
\]

Formula for compound interest over fixed periods within the year:

\[
A = P \left(1 + \frac{r}{n}\right)^{nt}
\]

A: Final amount.
P: Initial amount (Principal).
N: number of periods.
R: interest.
T: years.

Example:

Deposit $1000 at 12% interest
Paid monthly (n=12)
Leave it for 18 years (t=18)
How much will it be worth?

\[
A = 1000 \left(1 + \frac{0.12}{12}\right)^{12(18)}
\]

\[
= 1000 \left(1.01\right)^{216} =
\]

Non-IT Calculator \[y^x\] ① \[1.01\] then press ② \[y^x\] button
③ 216 ④ Enter \( \Rightarrow \) 857861
\[ A = 1000(1.0857861) = 8,578.61 \]

TI Calculator: \( \wedge \) exponent button

IN CLASS WORK

P 8.44

# 52) WHICH INSTITUTION PROVIDES THE BETTER INVESTMENT RATE?

A) FIDELITY SAYS "EARN 5.25% COMPOUNDED MONTHLY"

B) UNION SAYS "EARN 5.35% COMPOUNDED ANNUALLY"