GRAPHS OF INEQUALITIES

19. Graph the solution set of the linear inequality in one variable: $2x + 1 > 4$.

To graph the solution set of an inequality in one variable, we use a number line. To graph the solution set of an inequality in two variables, we use a rectangular coordinate system.

20. Graph the solution set of the linear inequality in two variables: $2x + y \geq 4$. See graphing answer section.

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CHAPTER REVIEW

SECTION 4.1

CONCEPTS

To solve an inequality, apply the properties of inequalities. If both sides of an inequality are multiplied (or divided) by a negative number, another inequality results, but with the opposite direction from the original inequality.

The graph of a set of real numbers that is a portion of a number line is called an interval.

Solving Linear Inequalities

REVIEW EXERCISES

Solve each inequality. Give each solution set in interval notation and graph it. See graphing answer section.

1. $5(x - 2) \leq 5 \to x \leq 3$
2. $0.3x - 0.4 \geq 1.2 - 0.1x$
3. $-16 < -\frac{4}{5}x \to x > 20$
4. $\frac{7}{4} (x + 3) < \frac{3}{8} (x - 3)$
5. $7 - [6t - 5(t - 3)] > 2(t - 3) - 3(t + 1)$
6. $\frac{2b + 7}{2} \leq \frac{3b - 1}{3}$

7. Explain how to use the graph of $y = 1$ and $y = x - 3$ to solve $x - 3 \leq 1$.

8. INVESTMENTS A woman has invested $10,000 at 6% annual interest. How much more must she invest at 7% so that her annual income is at least $2,000? $20,000 or more

SECTION 4.2

A solution of a compound inequality containing and makes both of the inequalities true.

Solving Compound Inequalities

Determine whether $-4$ is a solution of the compound inequality.

9. $x < 0 \text{ and } x > -5 \to \text{yes}$
10. $x + 3 < -3x - 1 \text{ and } 4x - 3 > 3x \to \text{no}$

Graph each set. See graphing answer section.

11. $(-3, 3) \cup [1, 6]$  
12. $(-\infty, 2] \cap [1, 4)$
Solve each compound inequality. Give the result in interval notation and graph the solution set. See graphing answer section.

13. \(-2x > 8 \text{ and } x + 4 \geq -6\) \((-10, -4]\)

14. \(5(x + 2) \leq 4(x + 1) \text{ and } 11 + x < 0\) \((-\infty, -4]\)

15. \(\frac{2}{5} x - 2 < -\frac{4}{5} \text{ and } \frac{x}{-3} < -1\) \(\emptyset\)

16. \(4\left(x - \frac{1}{4}\right) \leq 3x - 1 \text{ and } x \geq 0\) \([0, 0]\)

Solve each double inequality. Give the result in interval notation and graph the solution set. See graphing answer section.

17. \(3 < 3x + 4 < 10\) \((-\frac{1}{3}, 2]\)

18. \(-2 \leq \frac{5 - x}{2} \leq 2\) \([1, 7]\)

Determine whether \(-4\) is a solution of the compound inequality.

19. \(x < 1.6\) or \(x > -3.9\) yes

20. \(x + 1 < 2x - 1\) or \(4x - 3 > 3x\) no

Solve each compound inequality. Give the result in interval notation and graph the solution set. See graphing answer section.

21. \(x + 1 < -4\) or \(x - 4 > 0\) \((-\infty, -5) \cup (4, \infty)\)

22. \(x + 3 > -2\) or \(4 - x > 4\) \((2, \infty)\)

23. INTERIOR DECORATING A manufacturer makes a line of decorator rugs that are 4 feet wide and of varying lengths \(l\) (in feet). The floor area covered by the rugs ranges from 17 ft² to 25 ft². Write and then solve a double linear inequality to find the range of the lengths of the rugs. \(17 \leq 4l \leq 25\), \(4.25 \leq l \leq 6.25\) ft

24. Match each word in Column I with two items in Column II.

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. or ii. iv</td>
<td>i. (\cap)</td>
</tr>
<tr>
<td>b. and i. iii</td>
<td>ii. (\cup)</td>
</tr>
<tr>
<td>(\cap)</td>
<td>iii. intersection</td>
</tr>
<tr>
<td>(\cup)</td>
<td>iv. union</td>
</tr>
</tbody>
</table>

SECTION 4.3

Solving Absolute Value Equations and Inequalities

Solve each absolute value equation.

25. \(|4x| = 8\) \(x = 2, -2\)

26. \(2|3x + 1| - 1 = 19\) \(x = 3\)

27. \(\left|\frac{3}{2}x - 4\right| - 10 = -1\) \(26 \div 3 \cdot -10 = 3\)

28. \(\left|\frac{2 - x}{3}\right| = -4\) no solution

29. \(3x + 2 = |2x - 3|\) \(\frac{1}{5}, -5\)

30. \(\left|\frac{2(1 - x) + 1}{2}\right| = \left|\frac{3x - 2}{3}\right|\) \(x = \frac{13}{12}\)
Absolute value inequalities: For \( k > 0 \) and any algebraic expression \( X \):

\[ |X| < k \] is equivalent to \( -k < X < k \)

\[ |X| > k \] is equivalent to \( X < -k \) or \( X > k \)

31. \( |x| \leq 3 \quad [\text{Interval: } [-3, 3]] \)

32. \( 2x + 7 < 3 \)

33. \( 2|5 - 3x| \leq 28 \quad \left[\text{Interval: } [-3, \frac{19}{3}]\right] \)

34. \( \frac{2}{3}x + 14 > 6 < 6 \) \quad \text{no solution}

35. \( |x| > 1 \quad [\text{Interval: } (-\infty, -1) \cup (1, \infty)] \)

36. \( \frac{1 - 5x}{3} \geq 7 \)

37. \( 3x - 8 - 4 > 0 \quad \left[\text{Interval: } (-\infty, \frac{4}{3}] \cup (4, \infty)\right] \)

38. \( \frac{3}{2}x - 14 \geq 0 \quad [\text{Interval: } 1 - \infty, 14] \)

39. Explain why \( |0.04x - 8.8| < -2 \) has no solution. Since \( 0.04x - 8.8 \) is always greater than or equal to 0 for any real number \( x \), this absolute value inequality has no solution.

40. Explain why the solution set of \( \left| \frac{3x}{50} + \frac{1}{45} \right| \geq -\frac{4}{5} \) is the set of all real numbers. Since \( \frac{3x}{50} + \frac{1}{45} \) is always greater than or equal to 0 for any real number \( x \), this absolute value inequality is true for all real numbers.

41. PRODUCE Before packing, freshly picked tomatoes are weighed on the scale shown. Tomatoes having a weight \( w \) (in ounces) that falls within the highlighted range are sold to grocery stores.

a. Express this acceptable weight range using an absolute value inequality.

\[ w - 8 \geq 2 \]

b. Solve the inequality and express this range as an interval. [6, 10]

42. Let \( f(x) = \frac{1}{3} |6x| - 1 \). For what value(s) of \( x \) is \( f(x) = 5 \)? \( 3, -3 \)

SECTION 4.4

Linear Inequalities in Two Variables

Graph each inequality in the rectangular coordinate system. See graphing answer section.

43. \( 2x + 3y > 6 \)

44. \( y \leq 4 - x \)

45. \( y < \frac{1}{2}x \)

46. \( x \geq -\frac{3}{2} \)

47. CONCERT TICKETS Tickets to a concert cost $6 for reserved seats and $4 for general admission. If receipts must be at least $10,200 to meet expenses, find an inequality that shows the possible ways that the box office can sell reserved seats \( x \) and general admission tickets \( y \). Then graph the inequality for nonnegative values of \( x \) and \( y \) and give three ordered pairs that satisfy the inequality. \( 6x + 4y \geq 10,200 \); \( (1,800, 0), (1,000, 1,500), (2,000, 2,000) \)

48. Find the equation of the boundary line. Then give the inequality whose graph is shown. \( 3x - 4y \leq 12 \)
Systems of Linear Inequalities

Graph the solution set of each system of inequalities.

49. \[
\begin{align*}
 y &\geq x + 1 \\
3x + 2y &< 6
\end{align*}
\]

50. \[
\begin{align*}
 x - y &< 3 \\
y &\leq 0 \\
x &\geq 0
\end{align*}
\]

Graph each compound inequality in the rectangular coordinate system.

51. \[-2 < x < 4\]

52. \[y \leq -2 \text{ or } y > 1\]

53. PETROLEUM EXPLORATION
Organic matter converts to oil and gas within a specific range of temperature and depth called the petroleum window. The petroleum window shown can be described by a system of linear inequalities, where \(x\) is the temperature in °C of the soil at a depth of \(y\) meters. Determine what inequality symbol should be inserted in each blank.

\[
\begin{align*}
x &\geq 35 \\
x &\leq 130 \\
y &\geq -56x + 280 \\
y &\leq -18x + 90
\end{align*}
\]

54. In the illustration, the solution of one linear inequality is shaded in red, and the solution of a second is shaded in blue. Decide whether a true or false statement results if the coordinates of the given point are substituted into the given inequality.

a. \(A\), inequality 1 \hspace{1cm} true
b. \(A\), inequality 2 \hspace{1cm} false
c. \(B\), inequality 1 \hspace{1cm} true
d. \(B\), inequality 2 \hspace{1cm} false
e. \(C\), inequality 1 \hspace{1cm} true
f. \(C\), inequality 2 \hspace{1cm} true

**CHAPTER 4 TEST**

1. Decide whether the statement is true or false.
\(-5.67 \geq -5\) \hspace{1cm} false

2. Decide whether \(-2\) is a solution of the inequality.

\[3(x - 2) \leq 2(x + 7)\] \hspace{1cm} yes

3. \[7 < \frac{2}{3}x - 1\] \hspace{1cm} \((12, \infty)\)

4. \[-2(2x + 3) \geq 14\] \hspace{1cm} \((-\infty, -5]\)