Essential Question: Think about the question, but don't answer it. You will have an opportunity to answer this question at the end of the lecture.

**When adding integers, when do you get a positive answer? When do you get a negative answer?**

**Zero Pairs ( +, - )**
When you put one positive tile and one negative tile together, you have a zero pair. One might also look at this as one positive tile and one negative tile cancel each other out.

Notice that by using zero pairs, an integer can be represented with tiles in many different ways. Two examples are shown below.

**Example 1**
The integer +4 can be represented as

```
+ + + +
or
+ + + + + or
+ + + + + +
- 
```

**Example 2**
The integer -3 can be represented as

```
- - -
```

```
- - - - -
```

```
- - - -
```

```
+ +
```
Task 1: ADDITION
When you add tiles, think of the problem as combining all the tiles. Some zero pairs may result when you combine. Add the following integers and answer each question at the end using complete sentences.

a) $2 + 3$
2 positives plus 3 positives yields 5 positives.

\[
\begin{array}{cccccc}
+ & + & + & + & + & + \\
\end{array}
\]

\[
\begin{array}{cccccc}
+ & + & + & + & + & + \\
\end{array}
\]

ANSWER: _______

What is the sign result of adding two positive integers? Explain why

b) $-2 + (-4)$
2 negatives plus 4 negatives yields 6 negatives

\[
\begin{array}{cccc}
- & - & - & - \\
\end{array}
\]

\[
\begin{array}{cccc}
- & - & - & - \\
\end{array}
\]

Answer: _______

What is the sign result of adding two negative integers?

c) $4 + (-3)$
4 positives plus 3 negatives yields 3 zeros pairs and 1 positives

\[
\begin{array}{cccc}
+ & + & + & + \\
\end{array}
\]

\[
\begin{array}{cccc}
- & - & - \\
\end{array}
\]

\[
\begin{array}{cccc}
+ & + & + \\
- & - & - \\
\end{array}
\]

Answer: _______

What is the sign of the larger absolute value?
d) \(3 + (-5)\)

3 positives plus 5 negatives yields 3 zero pairs and 2 negatives.

\[
\begin{array}{cccc}
+ & + & + & - & - & - & - & - \\
+ & + & + & - & - & - & - & - \\
\end{array}
\]

Answer: 

What is the sign of the larger absolute value?

Task 2:
In your own words, what are some of the rules for adding integers?

Task 3:
Try the following problems. Use the positive symbol (+) for positive numbers and (-) for negative numbers.

a) \(4 + (-3)\)

b) \(3 + (-5)\)

c) \(-2 + 4\)

d) \(-3 + 2\)

e) \(-3 + 2 + (-5)\)
Task 4:
For the following two problems, apply the rules you came up with in Task 2

a) $52 + (-114)$

b) $12 + (-25) + 18 + (-13)$

Task 5: SUBTRACTION
When you subtract integers using the tile method, think of the problem as "take-away". If you do not have enough tiles to take away, ADD ZERO PAIRS, until you do have enough to take away.

a) $-5 - (-2)$

5 negatives take away 2 negatives yields __________.
(Fill in the blank. Don't forget to include the sign)

<table>
<thead>
<tr>
<th>Start with</th>
<th>Take away</th>
<th>Left over</th>
</tr>
</thead>
<tbody>
<tr>
<td>- - - - - -</td>
<td>- -</td>
<td>- - - -</td>
</tr>
</tbody>
</table>

Answer: ________
b) \(-2 - (-4)\)
2 negatives take away 4 negatives ... **DO NOT HAVE 4 NEGATIVES**, so we must add enough **zero pairs** to allow us to take 4 negatives.

This yields to ______________
(Fill in the blank. Don’t forget to include the sign)

<table>
<thead>
<tr>
<th>Start with</th>
<th>Add Zero Pairs</th>
<th>Take away</th>
<th>Left over</th>
</tr>
</thead>
<tbody>
<tr>
<td>- -</td>
<td>+ +</td>
<td>- - -</td>
<td>+ +</td>
</tr>
</tbody>
</table>

Answer: __________

Do you always get a negative answer when subtracting negative numbers? Explain why?

---

c) \(4 - (+2)\)
4 positives take away 2 positives.
This yields to ______________
(Fill in the blank. Don’t forget to include the sign)

<table>
<thead>
<tr>
<th>Start with</th>
<th>Take Away</th>
<th>Left over</th>
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</thead>
<tbody>
<tr>
<td>+ + + +</td>
<td>+ +</td>
<td>+ +</td>
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Answer: ________________________
d) $3 - (+5)$

3 positives take away 5 positives ..... **DO NOT have enough 5 positives** to take away, so we must add enough **zero pairs** to allow us to take 5 positives

<table>
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<th>Left over</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ + +</td>
<td>+ +</td>
<td>+ + + + +</td>
<td>- -</td>
</tr>
</tbody>
</table>

Answer: ____________

Do you always get a positive answer when subtracting positive numbers? Explain why.

---

e) $-3 - (+1)$

3 negatives take away 1 positive .... **DO NOT HAVE 1 positive to take away**, so we must add **enough zero pairs** to allow us to take 1 positive.

This yields _____________

<table>
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<th>Left over</th>
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<tbody>
<tr>
<td>- - -</td>
<td>+</td>
<td>+</td>
<td>- - - -</td>
</tr>
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</table>

Answer ____________
f) \( 2 - (-5) \)

2 positives take away 5 negatives ... **DO NOT HAVE 5 negatives** to take away, so we must add enough zero pairs to allow us to take 5 negatives.

This yields __________________

<table>
<thead>
<tr>
<th>Start with</th>
<th>Add zero pairs</th>
<th>Take away</th>
<th>Left over</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ +</td>
<td>+ + + + + +</td>
<td>- - - - -</td>
<td>+ + +</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+ + + + +</td>
<td></td>
</tr>
</tbody>
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Answer __________

When subtracting a positive and a negative, when do you get a positive answer? When do you get a positive answer?

---

**Task 6**

In your own words, what are some of the rules for subtracting integers?

---

**Task 7**

Try the following problems. Use the positive symbol (+) for positive numbers and (-) for negative numbers.

a) \( 6 - (-5) \)

b) \( -3 - (-3) \)

c) \( 2 - 3 \)
Task 8
For the following two problems, apply the rules you came up with in Task 7

a) $34 - (-21) - 45$

b) $(-112) - 234 - (435)$

Task 9
Answer the essential question

When adding integers, when do you get a positive answer? When do you get a negative answer?
2.2 ADDING INTEGERS "THE GANG RULE"

Let's assume the whole numbers are divided by zero (separated) into two "gangs": A) the crypts (+) and B) the bloods (-)

The addition problem

$5 + 3 = 8$ represents the fact that when 5 positives (+) join with 3 positives (+) (crypts) (crypts)
You get a party of 8 positives (+) = 8 crypts

Because "birds of a feather flock together," the crypts get along with each other allowing the size of the party (group) to increase without incident!

Similarly, the addition problem

$-3 + (-2) = -5$ represents the fact that when 3 negatives (-) join with 2 negatives (-) (bloods) (bloods)
You get a party of 5 negatives (-) = 5 bloods

Again, because "birds of a feather flock together," the bloods get along with each other, and the size of the party increases without incident!

When you add numbers from "different gangs", there is conflict!

The addition problem

$5 + (-3) = 2$ represents having 5 (+) crypts join with 3 (-) bloods
Since the gangs do not like each other, a violent fight erupts!
In the shootout, the 3 Bloods each take out a crypt, but are taken out themselves because they are "outnumbered," leaving 2 crypts that manage to survive!

This gang behavior is exactly the way signed numbers combine in addition problems. The basic rule is:

A) When numbers have the same sign, add their absolute values and give the sum the sign of the numbers.

B) When numbers have different signs, the "bigger gang" wins the battle, the answer is amount the bigger gang "outnumbered" the smaller gang.

Here are some more examples:

A) \(-6 + 2 = -4\)  \(\text{Negs outnumber the pos.}\)
B) \(14 + (-9) = 5\)  \(\text{Pos outnumber the neg.}\)
C) \(-2 + (-7) = -9\)  \(\text{Both are neg.}\)
D) \(-3 + (-11) = -8\)  \(\text{Neg outnumber the pos.}\)
E) \(-4 + 7 = 3\)  \(\text{Pos outnumber the neg.}\)
F) \(5 + 12 = 17\)  \(\text{Both are pos.}\)
2.3 Subtracting Integers

Subtraction is a myth, similar to "Santa Claus", "The Tooth Fairy", and many others.

Subtraction is really "Adding the Opposite Numbers". For example,

\[ 5 - 3 = 2 \] (Looks like \( +5 - (-3) \))

Really means
\[ +5 + (-3) = 2 \] (\( +5 + \text{neg } 3 \) = \( \text{the opposite of } 3 \))

\[ -7 - 4 = -11 \] (Looks like \( \text{neg } 7 - \text{neg } 4 \))

Really means
\[ -7 + (-4) = -11 \] (\( \text{neg } 7 + \text{neg } 4 \) = \( \text{the opposite of } 4 \))

\[ 4 - (-3) = 7 \] (Looks like \( 4 - \text{neg } 3 \))

Really means
\[ +4 + (-3) = 7 \] (\( 4 + \text{neg } 3 \) = \( \text{the opposite of } \text{neg } 3 \))

Here the two neg signs change to one pos sign for the 3 rule.

Simple Rule to Follow: When there is only one operator to the left,
A) Use the operator to the left of the number as the sign of the number and proceed like you are adding all the numbers.

B) If two (or more) signs are present, count the number of minus operators and if the count is odd make the number negative, if the count is even, make the number positive!

In Classwork

\[ \pm 52 \]
\[ 16 - 14 - (-9) = - \]
\[ \pm 60 \]
\[ (5 - 3) - (4 - 6) = - \]
ADDING MORE THAN TWO SIGNED NUMBERS

WHEN YOU HAVE MORE THAN TWO SIGNED NUMBERS, YOU CAN USE TWO DIFFERENT METHODS TO FIND THE SUM.
A) LEFT TO RIGHT (PEMDAS) OR B) BIRDS OF A FEATHER METHOD

A) PEMDAS TELLS US TO ADD OR SUBTRACT LEFT TO RIGHT, SO

\[
5 + (-8) + 6 + (-4) \; \text{DONE LEFT TO RIGHT} \\
= -3 + 6 = 3 + (-4) = -1
\]

However, we can also do the same problem using "Birds of a Feather"

\[
\Rightarrow \text{ADD all POS numbers} \Rightarrow 5 + 6 = 11 \\
\text{ADD all NEG numbers} \Rightarrow -8 + (-4) = -12 \\
\text{Now, add the two sums} \Rightarrow -12 + 11 = -1
\]

The reason "Birds of a Feather" works is due to the Commutative and Associative Properties of Addition.

IN CLASS WORK

PAGE 103

\#33) \(-15 + 8 = \) \\
\#46) \(-4 + 14 = \)

PAGE 104

\#57) \(9 + (-3) + 5 + (-4) = \)