Chapter 2.1 Primes, Composites, and Factors

New Terms:

Factor (verb): To write a product (multiply) of two numbers for a given number. Example: \( 10 = 5 \times 2 \)

Factor (noun): One of the numbers used in the product for a given number. Example: \( 10 = 5 \times 2 \), \( \Rightarrow \) 5 and 2 are factors of 10

Composite Number - A number that can be written as a product of two other factors that are smaller than it.

Examples:
\( 6 = 3 \times 2 \), \( 9 = 3 \times 3 \), \( 24 = 6 \times 4 = 2 \times 12 = 8 \times 3 \)

Prime Number - A number that cannot be written as a product of two other factors that are smaller than it. The only factors are itself and the number 1.

Examples:
\( 5 = 5 \times 1 \), \( 11 = 11 \times 1 \), \( 23 = 23 \times 1 \)

Factorization - one of the possible ways a number can be written as a product of two other numbers. For example: the factorizations of 24 are \( 2 \times 12 \), \( 6 \times 4 \), \( 8 \times 3 \), and \( 24 \times 1 \)

Prime Factor Tree - Writing a factorization of a number and continuing to factor each number until each composite number is broken up into prime numbers:

For example, the factor trees for the numbers 12 and 18 are

\[
\begin{align*}
12 & \quad 18 \\
/\ & \quad /\ \\
4 & \quad 6 \quad 3 \\
/\ & \quad /\ & \quad /\ \\
2 & \quad 2 & \quad 3 & \quad 3
\end{align*}
\]

Prime Factorization - The List of Primes you get at the bottom of your factor tree for a given number. Examples

\( 12 = 2 \times 2 \times 3 \)  
\( 18 = 2 \times 3 \times 3 \)

Later, we can use the Prime Factorization as a tool to help us reduce fractions:

Example: We want to reduce the fraction \( \frac{12}{18} = \frac{2(2)3}{2(3)3} \)

The common prime factors for numerator and denominator are depicted in red. If they are cancelled out of the fraction, the remaining factors create the reduced fraction:

\( \frac{2}{3} \)
In Class Problems:

1. Write the prime factorization for 36.

2. Write the prime factorization for 60.

Do Problems (a), (b)

2 (a) (b)

3 (a) (b)

From the following Activity Worksheet.
The Greatest Common Food

Team Name: ___________________
Your Name: _________________
Other team members: ________________________________

Task I: Prime Factoring
Each day at the GCF Prison, inmates are allowed to order breakfast. To make it a Secret, they order in code. The Code is a menu made from prime numbers:

<table>
<thead>
<tr>
<th>Prime Number</th>
<th>Food Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Glass of Water</td>
</tr>
<tr>
<td>2</td>
<td>Egg (Scrambled)</td>
</tr>
<tr>
<td>3</td>
<td>Bacon</td>
</tr>
<tr>
<td>5</td>
<td>Toast</td>
</tr>
<tr>
<td>7</td>
<td>Tea</td>
</tr>
<tr>
<td>11</td>
<td>Coffee</td>
</tr>
<tr>
<td>13</td>
<td>Orange Juice</td>
</tr>
</tbody>
</table>

Each order is a single number which reveals the items in the order when the number is broken down into its prime factors. For example, using the factor tree for the number 12 reveals that:

\[
12 \quad \begin{array}{c}
4 \\
\times 3 \\
2 \times 2 \times 3
\end{array}
\]

so the prime factors

So, a breakfast order of 12 = two 2’s and one 3, which is the order for two Eggs and one piece of Bacon.
1. What would the Code Numbers be to make the orders below?
   a. 1 Egg, 2 Bacon, 1 Toast, and 1 Tea.

       Answer__________________________

   b. 2 Egg, 1 Bacon, 1 Toast, and Tea.

       Answer__________________________

   c. 2 Toast, and 1 Tea.

       Answer__________________________

2. What would the Code Numbers below order from the menu?
   a. 30.

       Answer_______________________________________

   b. 42.

       Answer_______________________________________

   c. 66.

       Answer_______________________________________
**Task 2: Greatest Common Factor (GCF) Breakfast**

The mean Warden of the Prison decides to save money when the greedy prisoners order lots of different things by only giving them the items that each order has in common. One day two prisoners, Paris and Brittney order breakfast. Paris orders a code 12 but Brittney orders a code 18:

\[
\begin{align*}
12 & = 4 \cdot 3 \\
18 & = 6 \cdot 3 \\
2 \cdot 2 & \cdot 3 = 2 \cdot 3 \cdot 3
\end{align*}
\]

So, the Warden says since Paris ordered two Eggs (2’s) and one Bacon (3) while Brittney ordered one Egg (2) and two Bacon (3’s), what will be served to both of them is what the two orders have in common, which is one Egg (2) and one Bacon (3). So the Code for the breakfast that day is:

\[2 \cdot 3 = 6\text{ (one Egg and one Bacon)}\]

3. What would be the Code Number for the Prison Breakfast be if:
   a. Paris orders a Code 24 and Brittney orders a Code 32?

   **Answer**

   b. Paris orders a Code 35 and Brittney orders a Code 10?

   **Answer**

   c. Paris orders a Code 15 and Brittney orders a Code 6 and O.J (a new inmate) orders a Code 26?

   **Answer**