2.3 INTRO TO FRACTIONS

WE INTRODUCE FRACTIONS WITH THE DEFINITION
A FRACTION IS A RATIO (DIVISION) BETWEEN TWO WHOLE NUMBERS

\[
\frac{N}{D} \rightarrow \text{NUMERATOR (the count of the parts selected)}
\]

\[
\frac{D}{D} \rightarrow \text{DENOMINATOR (the number of equal parts to make one whole object)}
\]

We can use a diagram to illustrate fractions. For example,

\[
\frac{1}{3} = \begin{array}{c}
\text{\[\square\square\square\]} \\
\text{1 part out of} \\
\text{3 equal parts}
\end{array} \quad \frac{3}{5} = \begin{array}{c}
\text{\[\square\square\square\square\square\]} \\
\text{3 parts out of} \\
\text{5 equal parts}
\end{array}
\]

CLASSWORK

IDENTIFY THE FOLLOWING FRACTIONS:

(NEXT PAGE)
1. What part of the distance from 0 to 1 is shaded?

ANS __________

2. What part of the circle is shaded?

ANS __________

3. What part of the circle is shaded?

ANS __________
Mixed-Number Notation

Introduction...

If you are interested in the stock market, you know that, prior to the year 2000, stock prices were given in eighths. For example, on the day I wrote this introduction, one share of Intel Corporation was selling at $73 \frac{3}{8}$, or seventy-three and five-eighths dollars. The number $73 \frac{5}{8}$ is called a mixed number. It is the sum of a whole number and a proper fraction. With mixed-number notation, we leave out the addition sign.

Notation

A number such as $5 \frac{3}{4}$ is called a mixed number and is equal to $5 + \frac{3}{4}$. It is simply the sum of the whole number 5 and the proper fraction $\frac{3}{4}$, written without a + sign. Here are some further examples:

$$2 \frac{1}{8} = 2 + \frac{1}{8}, \quad 6 \frac{5}{9} = 6 + \frac{5}{9}, \quad 11 \frac{2}{3} = 11 + \frac{2}{3}$$

The notation used in writing mixed numbers (writing the whole number and the proper fraction next to each other) must always be interpreted as addition. It is a mistake to read $5\frac{3}{4}$ as meaning 5 times $\frac{3}{4}$. If we want to indicate multiplication, we must use parentheses or a multiplication symbol. That is:

$$5 \frac{3}{4} \text{ is not the same as } 5 \left( \frac{3}{4} \right)$$

USING DIAGRAMS WITH LINES WE CAN ILLUSTRATE MIXED NUMBERS

\[
\begin{array}{ccc}
\text{1 whole} & \text{1 whole} & \text{1/3} \\
\hline

\end{array}
\]

$$= 2 \frac{1}{3}$$

USING CIRCLES WE CAN ILLUSTRATE MIXED NUMBERS

\[
\begin{array}{ccc}

\end{array}
\]

$$= 2 \frac{3}{4}$$

CLASS WORK (NEXT PAGE)
CLASSWORK – IDENTIFY THE FOLLOWING AS MIXED NUMBERS

3. \[\begin{array}{c}
\text{Fraction Form} \quad \text{to} \quad \text{Mixed Form} \\
\frac{7}{2}
\end{array}\]
ANS _________

4.
\[\begin{array}{c}
\text{Fraction Form} \quad \text{to} \quad \text{Mixed Form} \\
\frac{19}{6} = 
\end{array}\]
ANS _________

5.
\[\begin{array}{c}
\text{Fraction Form} \quad \text{to} \quad \text{Mixed Form} \\
\frac{33}{10} = 
\end{array}\]
ANS _________
CHANGING MIXED NUMBERS TO IMPROPER FRACTIONS

EXAMPLE 2 Change $2\frac{1}{8}$ to an improper fraction.

\[
\begin{align*}
\text{SOLUTION} & \quad 2\frac{1}{8} = 2 + \frac{1}{8} \\
& = \frac{2}{1} + \frac{1}{8} \\
& = \frac{16}{8} + \frac{1}{8} \\
& = \frac{17}{8} \quad \text{Add the numerators}
\end{align*}
\]

Write as addition
Write the whole number 2 as a fraction
Change $\frac{2}{8}$ to a fraction with denominator 8

If we look closely at Examples 1 and 2, we can see a shortcut that will let us change a mixed number to an improper fraction without so many steps.

**Shortcut:** To change a mixed number to an improper fraction, simply multiply the denominator of the fraction part of the mixed number by the whole number, and add the result to the numerator of the fraction. The result is the numerator of the improper fraction we are looking for. The denominator is the same as the original denominator.

EXAMPLE 3 Use the shortcut to change $5\frac{3}{4}$ to an improper fraction.

\[
\begin{align*}
\text{SOLUTION} & \quad 1. \text{ First, we multiply } 4 \times 5 \text{ to get } 20. \\
& \quad 2. \text{ Next, we add } 20 \text{ to } 3 \text{ to get } 23. \\
& \quad 3. \text{ The improper fraction equal to } 5\frac{3}{4} \text{ is } \frac{23}{4}.
\end{align*}
\]

Here is a diagram showing what we have done:

\[
\begin{align*}
\text{Step 1} & \quad \text{Multiply } 4 \times 5 = 20. \\
& \quad \frac{5}{\frac{3}{4}} \\
\text{Step 2} & \quad \text{Add } 20 + 3 = 23.
\end{align*}
\]

Mathematically, our shortcut is written like this:

\[
\begin{align*}
5\frac{3}{4} & = \frac{(4 \cdot 5) + 3}{4} = \frac{20 + 3}{4} = \frac{23}{4}
\end{align*}
\]

The result will always have the same denominator as the original mixed number.
**EXAMPLE 4** Change $6 \frac{5}{9}$ to an improper fraction.

**SOLUTION** Using the first method, we have

$$6 \frac{5}{9} = 6 + \frac{5}{9} = \frac{6}{1} + \frac{5}{9} = \frac{9 \cdot 6}{9} + \frac{5}{9} = \frac{54}{9} + \frac{5}{9} = \frac{59}{9}$$

Using the shortcut method, we have

$$6 \frac{5}{9} = \frac{(9 \cdot 6) + 5}{9} = \frac{54 + 5}{9} = \frac{59}{9}$$

**Changing Improper Fractions to Mixed Numbers**

To change an improper fraction to a mixed number, we divide the numerator by the denominator. The result is used to write the mixed number.

**EXAMPLE 5** Change $\frac{11}{4}$ to a mixed number.

**SOLUTION** Dividing 11 by 4 gives us

$$\begin{array}{c}
2 \\
\frac{8}{3}
\end{array}$$

We see that 4 goes into 11 two times with 3 for a remainder. We write this result as

$$\frac{11}{4} = 2 + \frac{3}{4} = 2 \frac{3}{4}$$

The improper fraction $\frac{11}{4}$ is equivalent to the mixed number $2 \frac{3}{4}$.

An easy way to visualize the results in Example 5 is to imagine having 11 quarters. Your 11 quarters are equivalent to $\frac{11}{4}$ dollars. In dollars, your quarters are worth 2 dollars plus 3 quarters, or $2 \frac{3}{4}$ dollars.

---

**EXAMPLE 6**

$$\begin{array}{c}
10 \\
3
\end{array} \Rightarrow \frac{10}{3} = 3 + \frac{1}{3} = 3 \frac{1}{3}$$

**EXAMPLE 7**

$$\begin{array}{c}
208 \\
24
\end{array} \Rightarrow 8 \Rightarrow \frac{208}{24} = 8 + \frac{16}{24} = 8 + \frac{2}{3} = 8 \frac{2}{3}$$

Reduce to lowest terms
In Class Work

7) Change $3\frac{1}{4}$ to an Improper Fraction

8) Change $\frac{22}{5}$ to a Mixed Number

9) Change $\frac{43}{12}$ to a Mixed Number
2.4 REDUCING FRACTIONS
EQUIVALENT FRACTIONS
AND LOWEST TERMS

If two fractions represent the same amount, they are called "equivalent fractions." For example

\[
\frac{4}{6} \quad \text{and} \quad \frac{2}{3}
\]

are the same amount as shown by the diagrams. We can also show it mathematically with the cancel rule:

\[
\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{3}
\]

Another Example

\[
\frac{12}{15} = \frac{3 \cdot 4}{3 \cdot 5} = \frac{4}{5}
\]

It is ok to use multiple steps to find the simplest form (Lowest Denominator). For example

\[
\frac{18}{24} = \frac{2 \cdot 9}{2 \cdot 12} = \frac{9}{12} = \frac{3 \cdot 3}{3 \cdot 4} = \frac{3}{4}
\]

But we could have done it in one step with a common factor of "6"

\[
\frac{18}{24} = \frac{3 \cdot 6}{6 \cdot 4} = \frac{3}{4}
\]

This is the GCF

Both have 6
CLASSWORK 2.4

1) Reduce the fraction (same as write in lowest terms)
\[
\frac{21}{35}
\]

2) Are the two fractions equivalent? (Yes or No)
\[
\frac{12}{15}, \quad \frac{28}{35}
\]