The first example below shows that fractions could be used in place of the decimals, but it makes the problem "hard" when it is actually "very easy." All we need to do is count the total number of Decimals in the two numbers we are multiplying. The answer must have the same number of Decimals as the "Total".

Looking over these first two examples, we can see that the digits in the result are just what we would get if we simply forgot about the decimal points and multiplied, that is, \(3 \times 5 = 15\). The decimal point in the result is placed so that the total number of digits to its right is the same as the total number of digits to the right of both decimal points in the original two numbers. The reason this is true becomes clear when we look at the denominators after we have changed from decimals to fractions.

**EXAMPLE 3**

Multiply: \(2.1 \times 0.07\)

**SOLUTION**

\[
2.1 \times 0.07 = \frac{21}{10} \times \frac{7}{100} = \frac{21 \times 7}{10 \times 100} = \frac{147}{1000} = 0.147
\]

Again, the digits in the answer come from multiplying \(21 \times 7 = 147\). The decimal point is placed so that there are three digits to its right, because that is the total number of digits to the right of the decimal points in 2.1 and 0.07.

We summarize this discussion with a rule.

**RULE**

To multiply two decimal numbers:

1. Multiply as you would if the decimal points were not there.
2. Place the decimal point in the answer so that the number of digits to its right is equal to the total number of digits to the right of the decimal points in the original two numbers in the problem.

**EXAMPLE 4**

How many digits will be to the right of the decimal point in the following product?

\(3.706 \times 55.88\)

**SOLUTION**

There are three digits to the right of the decimal point in 2.987 and two digits to the right in 24.82. Therefore, there will be \(3 + 2 = 5\) digits to the right of the decimal point in their product.

Answers

2. 0.0035 3. 0.14 4. 5
EXAMPLE 5 Multiply: $3.05 \times 4.36$

SOLUTION We can set this up as if it were a multiplication problem with whole numbers. We multiply and then place the decimal point in the correct position in the answer.

\[
\begin{array}{c}
3.05 \\
\times 4.36 \\
\hline
1830 \\
915 \\
1220 \\
\hline
132980
\end{array}
\]

\[
\begin{array}{c}
\text{2 digits to the right of decimal point} \\
\hline
\text{2 digits to the right of decimal point}
\end{array}
\]

---

The decimal point is placed so that there are \(2 + 2 = 4\) digits to its right.

As you can see, multiplying decimal numbers is just like multiplying whole numbers, except that we must place the decimal point in the result in the correct position.

Estimating

Look back to Example 5. We could have placed the decimal point in the answer by rounding the two numbers to the nearest whole number and then multiplying them. Because 3.05 rounds to 3 and 4.36 rounds to 4, and the product of 3 and 4 is 12, we estimate that the answer to $3.05 \times 4.36$ will be close to 12. We then place the decimal point in the product 132980 between the 3 and the 2 in order to make it into a number close to 12.

EXAMPLE 6 Estimate the answer to each of the following products.

a. $29.4 \times 8.2$  
b. $68.5 \times 172$  
c. $(6.32)^2$

SOLUTION

a. Because 29.4 is approximately 30 and 8.2 is approximately 8, we estimate this product to be about $30 \times 8 = 240$. (If we were to multiply 29.4 and 8.2, we would find the product to be exactly 241.08.)

b. Rounding 68.5 to 70 and 172 to 170, we estimate this product to be $70 \times 170 = 11,900$. (The exact answer is 11,782.) Note here that we do not always round the numbers to the nearest whole number when making estimates. The idea is to round to numbers that will be easy to multiply.

c. Because 6.32 is approximately 6 and $6^2 = 36$, we estimate our answer to be close to 36. (The actual answer is 39.9424.)

IN CLASS WORK #1

Example 6 #(b) $28.3 \times 0.59$

---

Answers

5. $21.0366$

6. a. 480  
b. 7,200  
c. 64
DIVISION WITH DECIMALS

**RULE**
To divide a decimal by a whole number, we do the usual long division as if there were no decimal point involved. The decimal point in the answer is placed directly above the decimal point in the problem.

Here are some more examples to illustrate the procedure.

**EXAMPLE 3** Divide: 49.896 ÷ 27

**SOLUTION**

\[
\begin{array}{c|c}
27 & 49.896 \\
22 & \\
21 & 6 \\
1 & 29 \\
1 & 08 \\
2 & 16 \\
2 & 16 \\
0 & \\
\end{array}
\]

Check this result by multiplication:

\[
\begin{array}{c|c}
27 & 1.848 \\
\times & 27 \\
\hline
1 & 29 \\
3 & 696 \\
4 & 983 \\
0 & \\
\end{array}
\]

We can write as many zeros as we choose after the rightmost digit in a decimal number without changing the value of the number. For example,

\[
6.91 = 6.910 = 6.9100 = 6.91000
\]

There are times when this can be very useful, as Example 4 shows.

**EXAMPLE 4** Divide: 1,138.9 ÷ 35

**SOLUTION**

\[
\begin{array}{c|c}
35 & 1,138.9 \\
1 & 05 \\
8 & 8 \\
7 & 0 \\
1 & 29 \\
1 & 40 \\
1 & 40 \\
0 & 1,138.9 \\
\end{array}
\]

Write 0 after the 9. It doesn't change the original number, but it gives us another digit to bring down.

Check:

\[
\begin{array}{c|c}
32.54 & \\
\times & 35 \\
\hline
1 & 40 \\
9 & 762 \\
0 & 1,138.9 \\
\end{array}
\]

Until now we have considered only division by whole numbers. Extending division to include division by decimal numbers is a matter of knowing what to do about the decimal point in the divisor.

**EXAMPLE 5** Divide: 31.35 ÷ 3.8

**SOLUTION**

In fraction form, this problem is equivalent to

\[
\frac{31.35}{3.8}
\]

If we want to write the divisor as a whole number, we can multiply the numerator and the denominator of this fraction by 10:

\[
\frac{31.35 \times 10}{3.8 \times 10} = \frac{313.5}{38}
\]

Answers

3. 2.636
4. 45.54
So, since this fraction is equivalent to the original fraction, our original division problem is equivalent to

\[
\begin{array}{c}
8.25 \\
38317.50 \\
\hline
304 \\
95 \\
76 \\
190 \\
190 \\
\hline
0
\end{array}
\]

Put 0 after the last digit

We can summarize division with decimal numbers by listing the following points, as illustrated by the first five examples.

### Summary of Division with Decimals

1. We divide decimal numbers by the same process used in Chapter 1 to divide whole numbers. The decimal point in the answer is placed directly above the decimal point in the dividend.
2. We are free to write as many zeros after the last digit in a decimal number as we need.
3. If the divisor is a decimal, we can change it to a whole number by moving the decimal point to the right as many places as necessary so long as we move the decimal point in the dividend the same number of places.

### Example 6

Divide, and round the answer to the nearest hundredth:

\[
0.3778 \div 0.25
\]

**Solution**

First, we move the decimal point two places to the right:

\[
0.25 \times 37.78 = 9.445
\]

Then we divide, using long division:

\[
\begin{array}{c}
1.5112 \\
25 \overline{37.7800} \\
25 \downarrow \\
127 \\
125 \downarrow \\
28 \\
25 \downarrow \\
30 \\
25 \downarrow \\
50 \\
50 \downarrow \\
0
\end{array}
\]

Rounding to the nearest hundredth, we have 1.51. We actually did not need to have this many digits to round to the hundredths column. We could have stopped at the thousandths column and rounded off.

### Note

We do not always use the rules for rounding numbers to make estimates. For example, to estimate the answer to Example 5, \(31.35 \div 3.8\), we can get a rough estimate of the answer by reasoning that 3.8 is close to 4 and 31.35 is close to 32. Therefore, our answer will be approximately \(32 \div 4 = 8\).

### Example 6

Divide, and round your answer to the nearest hundredth:

\[
0.4553 \div 0.32
\]

**Note**

Moving the decimal point two places in both the divisor and the dividend is justified like this:

\[
\begin{array}{c}
0.3778 \times 100 = 37.78 \\
0.25 \times 100 = 25
\end{array}
\]

### Answers

5. 3.15 6. 1.42

*Taken from "Basic Mathematics," C. McKeague, 6th edition, Thomson/Brooks/Cole
IN CLASS WORK #2

pg 349 #24)  Divide

0.52 \sqrt{3.2318}