METHODS OF INTEGRATION
Keep this sheet handy for the rest of your life.

1) To integrate $x^n e^{ax}$, $x^n \sin(ax)$, or $x^n \cos(ax)$ where $n$ is a positive integer and $a$ is a constant, integrate by parts $n$ times, differentiating the power of $x$ and integrating the second part.

2) To integrate $x^n \ln x$, where $n$ is an integer, integrate by parts, differentiating the logarithmic term.

3) To integrate products of $e^{ax}$, $\sin(bx)$, $\cos(cx)$, where $a, b, &c$ are constants, integrate by parts twice. This will yield an equation that can be solved for the original integral.

4) To integrate $\sin^n x \cos^m x$:
   - **If $n$ is odd**: Change all but one sine to cosines using $\sin^2 x = 1 - \cos^2 x$, then use the substitution $u = \cos x$.
   - **If $m$ is odd**: Change all but one cosine to sines using $\cos^2 x = 1 - \sin^2 x$, then use the substitution $u = \sin x$.
   - **If both $m$ and $n$ are even**: Express the integrand in terms of either cosines or sines (but not both) using $\sin^2 x + \cos^2 x = 1$ or $\sin^2 x = \frac{1}{2} (1 - \cos 2x)$, then integrate directly where possible or use repeated integration by parts.

5) To integrate $\sin(ax)\sin(bx)$, $\sin(ax)\cos(bx)$, or $\cos(ax)\cos(bx)$, use one of the following:
   - $\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$
   - $\cos A \cos B = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$
   - $\sin A \cos B = \frac{1}{2} [\sin(A-B) + \sin(A+B)]$.

6) To integrate $\tan^m x \sec^n x$:
   - **If $m$ is odd**: Make the substitution $u = \sec x$. Change all but one tangent term to secant using the identity $\tan^2 x = \sec^2 x - 1$.
   - **If $n$ is even**: Make the substitution $u = \tan x$. Change all but two secant terms to tangent using the identity $\sec^2 x = \tan^2 x + 1$.
   - **If $m$ is even and $n$ is odd**: Change all of the tangent terms to secant and integrate using repeated integration by parts.
7) Integrals containing $\sqrt{a^2 - x^2}$, $\sqrt{x^2 + a^2}$, or $\sqrt{x^2 - a^2}$ can be integrated using the following triangles to make trigonometric substitutions:

\[ x = \sin \theta \]
\[ x = \tan \theta \]
\[ x = \sec \theta \]

8) For integrals with quadratics $(ax^2 + bx + c)$ one should complete the square. If you are lucky, the resulting expression will be integrable (though maybe not in a single step).