Example: Page 150 #6:

Show that the vector space of all convergent infinite sequences is not finite-dimensional

[Note: The statement of the problem indicates that the set of all convergent infinite sequences is a vector space, so we do not have to prove that.]

Let us name the vector space of all convergent infinite sequences something a bit more friendly, say Dave.

Let $S_i = \{a_k\}$ where $a_k = \begin{cases} 0 & \text{if } k \neq i \\ 1 & \text{if } k = i \end{cases}$

Thus $S_1 = \{1, 0, 0, 0, \ldots \}$
$S_2 = \{0, 1, 0, 0, \ldots \}$
$S_3 = \{0, 0, 1, 0, \ldots \}$ etc.

The set $\{S_1, S_2, S_3, S_4, S_5, \ldots, S_n\}$ contains $n$ elements which are independent. Thus $\dim(Dave) \geq n$. However, for any finite value of $n$, the set $\{S_1, S_2, S_3, S_4, S_5, \ldots, S_n, S_{n+1}\}$ contains $n+1$ independent elements and so $\dim(Dave) \geq n + 1 > n$.

Thus for any finite $n$, $\dim(Dave) > n$, hence $\dim(Dave)$ is not finite.