

## Chapter 5 - Probability

**What's the likelihood of something occurring?** Can we answer questions about probabilities using data or experiments? For instance:

- 1) If my parking meter expires, I will probably get a parking ticket.

Say you want to find out how likely it is that a person will get a parking ticket within 30 minutes of the expiration of the parking meter. You ask all of your friends to take a risk and let their parking meter expire and then wait 30 minutes to see if a parking ticket is written. Of your 50 friends, 8 received a parking ticket. The **experimental probability** of getting a parking ticket is

$$P(\text{getting a parking ticket}) = 8/50.$$

The number  $8/50$  is called the relative frequency.

- 2) There is no chance I will pass tomorrow's quiz.

Could we reasonably put a number to what it means to have no chance of passing a quiz. Well, suppose the REASON there is no chance of passing the quiz is because the instructor writes quizzes that are bloody difficult. Somehow, you have access to the instructor's records for this quiz, which she has given every semester for 25 years. Of the 2387 students who took this quiz over the year, you find that 91 students have passed the quiz.

What is the relative frequency of passing this quiz?

$$P(\text{passing this quiz}) = \boxed{\phantom{0000}}.$$

- 3) Tom, a manager of a small apartment complex, recently replaced all the doorbells on all of the units. Based on his experience, about  $\frac{1}{4}$  of the doorbells will break before six months pass. Tom turns to the manufacturer to get answers. The company tells him that 3% of these doorbells become defective within sixth months of installation. How do you suppose the manufacture reached the figure 3%?
- 4) Joyce told me that moving to Seattle in the next two years is next to impossible. Could we possibly find or produce data or conduct any experiment to see what Joyce might mean by "next to impossible" or to contradict Joyce's opinion?

**Definitions to know:**

**a) Probability** - The quantitative (numerical) measure of something occurring. There are 3 types of probability.....

- **Classical Probability** - Probability using mathematical logic.

$$P(E) = \frac{\text{number of ways } E \text{ can occur}}{\text{size of sample space}}$$

- **Empirical probability (Frequency)** - Probability is measured by using the results of an experiment.
  
- **Subjective probability** - Probability using history and estimating.

**Example #1**

Let us use the procedure (or experiment) of tossing a coin to get comfortable with some definitions.

- ◆ Experiment: Toss a coin
  
- ◆ What can happen? **(This is called an event)**
  
- ◆ How many events are there?
  
- ◆ Make a list of all possible events. **(This is called the sample space)**

## Definitions Continued...

Ok ... Now considered the following EXPERIMENTS...

- a) Drawing a card from a regular 52 deck
- b) Rolling a die

b) **Event** - the outcome of a procedure or an experiment.

c) **Simple event** - one outcome

d) **Compound event** - more than one outcome

e) **Sample space** - The set of all possible outcomes of a random experiment is known as the sample space and is denoted by  $S$ .

**f) Probabilities Rules (PROPERTIES!!)**

Using the "rolling a die" experiment, apply the properties, to help you understand what they mean.

- $0 \leq P(E) \leq 1$

- $P(E) = 0$

- $P(E) = 1$

- $\sum P(E) = 1$

- $P(E) + P(\overline{E}) = 1$

**Example #1 - Construct a Tree Diagram**

If a couple has three children family, what is the probability of having exactly 2 girls?

All boys?

All girls?

**Example #2**

Describe the sample space for the following experiments.

**a) #22 in your book**

If a basketball player shoots three free throws, describe the sample of possible outcomes using  $S$  for a free throw and  $F$  for a missed free throw.

**b) Tossing 2 coins 3 times**

## Section 5.2 - Addition Rule for Probability

### Definitions to know

a) Mutually Exclusive Events ( disjoint events)

b) Addition Rule for mutually exclusive events

$$P(A \text{ or } B) = P(A) + P(B)$$

*"The probability that in a SINGLE event, event A occurs or event B occurs"*

c) Addition Rule for NOT mutually exclusive events

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

*"The probability that in a SINGLE event, event A occurs or event B occurs or they both occur"*

### Example #1

Determine whether these events are mutually exclusive.

a) Roll a die: Get an even number, and get a number less than 3.

b) Roll a die: Get a number greater than 3, and get a number less than 3.

c) Select a student in your college: The student is a sophomore and the student is a business major.

**Example #2**

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$E = \{2, 3, 4, 5, 6, 7\}$$

$$F = \{5, 6, 7, 8, 9\}$$

$$G = \{9, 10, 11, 12\}$$

$$H = \{2, 3, 4\}$$

a) List the outcomes in  $E$  and  $F$ . Are these two events mutually exclusive?

b) List the outcomes in  $E$  or  $H$ .

c) Now find  $P(E \text{ or } H)$  by counting the number of outcomes in  $E$  or  $H$ .

d) Determine  $P(E \text{ or } H)$  using the Addition Rule.

e) List the outcomes in  $F$  and  $H$ . Are  $F$  and  $H$  mutually exclusive? Explain why?

f) List the outcomes in  $F^c$ . Find  $P(F^c)$

**Example #2**

$$P(E) = 0.25$$

$$P(F) = 0.45$$

a) Find  $P(E^c)$

b) Find  $P(E \text{ and } F)$  if  $P(E \text{ or } F) = 0.6$

**Example #3**

At a community swimming pool there are 2 managers, 8 lifeguards, 3 concession stand clerks, and 2 maintenance people. If a person is selected at random, find the probability that a person is either a lifeguard or a manager.

**Example #4**

At a particular school with 200 male students, 58 play football, 40 play basketball, and 8 play both.

a) Construct a Venn Diagram

b) What is the probability that a randomly selected male student plays a sport (football or basketball)?

c) What is the probability that a randomly selected male student plays neither sport?

**Example #5**

In a statistics class there are 18 juniors and 10 seniors; 6 of the seniors are females and 12 of the juniors are males. If a student is selected at random, find the probability of selecting the following

a) A junior or a female

b) A senior or a female

c) A junior or a senior

## At Least Problems - Using the complement.

$$P(E) = 1 - P(\bar{E})$$

### Example #6

With one method of sampling, a sample of items is randomly selected without replacement, and the entire batch is rejected if there is at **least one defect**. The Medtyme Pharmaceutical Company has just manufactured 2500 aspirin tablets, and 2% are defective because they contain too much or too little aspirin. If 4 of the tablets are selected and tested, what is the probability that the entire batch will be rejected?

### Example #7

If you make random guesses for four multiple-choice test questions (each with five possible answers), what is the probability of getting at least one correct? If a non-demanding instructor says that passing the test occurs if there is **at least one** correct answer, can you reasonably expect to pass by guessing?



**Example #10**

The following data represents the number of live multiple-delivery births (three or more babies) in 2002 for women 15 to 54 years old:

Age	Number of Multiple Births
15-19	93
20-24	511
25-29	1628
30-34	2832
35-39	1843
40-44	377
45-54	117

- a) Determine the probability that a randomly selected multiple birth in 2002 for women 15 to 54 years old involved a mother 30 to 39 years old.
- b) Determine the probability that a randomly selected multiple birth in 2002 for women 15 to 54 years old who was not 30 to 39 years old.
- c) Determine the probability that a randomly selected multiple birth in 2002 for women 15 to 54 years old involved a mother who was less than 45 years old.
- d) Determine the probability that a randomly selected multiple birth in 2002 for women 15 to 54 years old involved a mother who was at least 20 years old.

## Section 5.3 and Section 5.4 - The Multiplication Rules and Conditional Probability

Definitions to know:

a) Independent Events

b) Dependent events

c) Multiplication Rule for independent events

$$P(A \text{ and } B) = P(A)P(B)$$

"The probability that event A occurs in a first trial and event B occurs in a second trial"

d) Conditional Probability

e) Multiplication Rule for dependent event

$$P(A \text{ and } B) = P(A)P(B/A)$$

f) Formula for Conditional Probability

$$P(B/A) = \frac{P(A \text{ and } B)}{P(A)}$$

**Example #1** - Are the following events independent or dependent?

a) Tossing a coin; Rolling a die

b) Drinking or using drugs until your driving ability is impaired ; Being involved in a car crash

**Example #2**

If 2 cards are selected from a standard deck of 52 cards with REPLACEMENT, find the following probabilities

a) Both are red cards

b) Both are kings

**Example #3**

If 2 cards are selected from a standard deck of 52 cards without replacement, find the following probabilities

a) One is jack and the second is a queen

b) Both are spades

c) Both are kings

**Example #4**

A quick quiz consists of a true/false question followed by a multiple choice question with four possible answers (a, b, c, d). If both questions are answered with random guesses, find the probability that both responses are correct. Does guessing appear to be a good strategy on this quiz?

**Example #5**

Consider the following table of 100 Senators from the 108<sup>th</sup> Congress of the United States.

	Republican	Democrat	Independent
Male	46	39	1
Female	5	9	0

- a) If we randomly select one Senator, what is the probability of getting a Republican, given that a male was selected?
- b) If we randomly select one Senator, what is the probability of getting a male, given that a Republican was selected? Is this the same result found above?



## Section 5.5 - Counting Techniques

### Definitions to know:

#### a) The fundamental Counting Rule (Multiplication Rule)

For a sequence of TWO events in which the first event can occur  $m$  ways and the second event can occur  $n$  ways, the events together can occur a total of  $(m)(n)$  ways.

#### b) Factorial Formulas

$$0! = 1$$

$$n! = (n)(n-1)(n-2)\cdots(2)(1)$$

c) **Permutation** - an arrangement of  $n$  objects in a specific order.

#### d) Permutation Rule

There are  $n$  different items available.

We select  $r$  of the  $n$  items (without replacement)

We consider rearrangements of the same items to be different sequences.

$${}_n P_r = \frac{n!}{(n-r)!}$$

The number of permutations of  $r$  items from  $n$  different available items

e) **Combination** - A section of distinct object without regard to order

#### f) Combination Rule

There are  $n$  different items available

We select  $r$  of the  $n$  items (without replacement)

We consider rearrangements of the same items to the same.

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

The number of combinations of  $r$  items selected from  $n$  different item

**Example #1**

How many different ways can 9 trophies be arranged on a shelf?

**Example #2**

How many different ID cards can be made if there are 6 digits on a card and no digit can be used more than once?

What if digits ARE allowed more than once, (**Repetition is allowed!!**)

**Example #3**

How many ways can there be to select 3 bracelets from a box of 10 bracelets, disregarding order?

**Example #4**

How many ways can a committee of 4 people be selected from a group of 10 people?

**Example #5**

How many ways can a jury of 6 women and 6 men be selected from 10 women and 12 men?

**Example #6**

A committee of 4 people is to be formed from 6 doctors and 8 dentist. Find the probability that the committee will consist of

a) All dentist

---

b) 2 dentist and 2 doctors

---

c) All doctors

---

d) 3 doctors and 1 dentist

---

e) 1 doctor and 3 dentist

---

f) John, Sam, Eric and Jake

---

**Example #7**

At a sorority meeting, there are 6 seniors, 4 juniors and 2 sophomores.

- a) If a committee of 3 is to be selected, how many different arrangements are there for the committee?
  
  
  
  
  
  
  
  
  
  
- b) What is the probability that 1 of each will be selected.
  
  
  
  
  
  
  
  
  
  
- c) Now, if a committee consists of the PRESIDENT, VICE PRESIDENT, and SECRETARY, how many different arrangements are there?
  
  
  
  
  
  
  
  
  
  
- d) What is the probability that a senior will be selected as PRESIDENT, a junior will be selected as VICE PRESIDENT and that a sophomore will be selected as SECRETARY?
  
  
  
  
  
  
  
  
  
  
- e) Sara, Jamie and Jane are best friends and belong to this sorority. What is the probability that Sara will be selected as PRESIDENT, Jamie will be selected as VICE PRESIDENT and Jane as SECRETARY?

**Example #8**

On a lunch counter there are 3 oranges, 5 apples, and 2 bananas. If 3 pieces of fruit are selected, find the probability that 1 orange, 1 apple and 1 banana are selected.



## Section 5.6 - Putting It Together: Which Method Do I Use?

### Problem 1

If you draw an M&M candy at random from a bag of the candies, the candy you draw will have one of six colors. The probability of drawing each color depends on the proportion of each color among all candies made. Here is the distribution for milk chocolate M&M's:

Color	Yellow	Red	Orange	Brown	Green	Blue
Probability	0.14	0.13	0.20	0.13	0.16	?

- What must be the probability of drawing a blue candy?
- What is the probability that you do not draw a brown candy?
- What is the probability that the candy you draw is either yellow, orange, or red?

### Problem 2 - Classifying occupations

Choose an American worker at random and classify his or her occupation into one of the following classes. These classes are used in government employment data.

- A - Managerial and professional
- B - Technical, sales, administrative support
- C - Service occupations
- D - Precision production, craft, and repair
- E - Operators, fabricators, and laborers
- F - Farming, forestry, and fishing

The table below gives the probabilities that a randomly chosen worker falls into each of 12 sex-by-occupation classes:

Class	A	B	C	D	E	F
Male	0.14	0.11	0.06	0.11	0.12	0.03
Female	0.09	0.20	0.08	0.01	0.04	0.01

- Verify that this is a legitimate assignment of probabilities to these outcomes.
- What is the probability that the worker is female?
- What is the probability that the worker is not engaged in farming, forestry, or fishing?
- Classes D and E include most mechanical and factory jobs. What is the probability that the worker holds a job in one of these classes?
- What is the probability that the worker does not hold a job in Classes D or E?