Chapter 7 - Normal Probability Distribution.

- When the data values are evenly distributed about the mean, the distribution is said to be symmetrical.
- When the majority of the data values fall to the left or right of the mean, the distribution is said to be skewed. It is called right skewed (Positively Skewed) if the mean falls to the right of the median, and it is called left skewed (negatively Skewed) if the mean falls to the left of the median.

Section 7.1 and 7.2
The Standard Normal Distribution
Definitions to know:

a) Probability density function

\[
y(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

b) Normal distribution
d) Uniform distribution

- Find $P(x > 20)$
- Find $P(x \geq 20)$
- Find $P(20 \leq x < 35)$
- Find $P(x < 20)$

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e) Standard normal distribution
Example #1
Using the standard normal distribution, first make a drawing of the probabilities and find the probabilities.

a) \( P(0 < z < 1.87) \)

b) \( P(-1.23 < z < 0) \)

c) \( P(1.12 < z < 1.43) \)

d) \( P(z < -1.42) \)

e) \( P(-1.42 < z < 1.42) \)
Example #2
Find the z-value that corresponds to the given area.

Section 7.3
Applications of the Normal distribution

Example 1
The average admission charge for a movie is $5.39. If the distribution of admission charges is normal with a standard deviation of $0.79, what is the probability that a randomly selected admission charge is less than $3?
Example 2
The national average SAT score is 1019. If we assume a normal distribution with $\sigma = 90$.

a) What is the 90th percentile score?

b) What is the probability that a randomly selected score exceeds 1200?

c) Suppose my nephew wants to attend a TOP university where ONLY the top 5% scores are admitted. What score should my niece score on the SAT attend this university.

Example #3
For a medical study, a researcher wishes to select people in the middle 60% of the population based on blood pressure. If the mean blood pressure is 120 and the standard deviation is 8, find the upper and lower reading that would qualify people to participate in the study.
Example #4 (Number 26 in your book)
Ball bearings are manufactured with a mean diameter of 5mm. Because of variability in the manufacturing process, the diameters of the ball bearings are approximately normally distributed with a standard deviation of 0.02 mm.

a) What proportion of ball bearings has a diameter more than 5.03 mm?

b) Any ball bearings that have a diameter less than 4.95mm or greater than 5.05mm are discarded. What proportion of ball bearings will be discarded?

c) Using results of part b), if any 30,000 ball bearings are manufactured in a day, how many should the plant manager expect to discard?

d) If an order comes in from 50,000 ball bearings, how many bearings should the plant manager manufacture if the order states that all ball bearings must be between 4.97mm and 5.03mm?
Section 7.4 - Assessing Normality

How do we know when the sample we've taken is normally distributed? In this class, most of our calculations require that the population that the sample has been taken is NORMALLY DISTRIBUTED.

For large samples, we can easily (using technology or by hand) construct a graph that can help us determine its populations distribution.

- Many real world variables have bell shaped histograms, so we would say that they should or could have normal probability distributions
- We need methods to assess whether this is a good assumption or not

For a small sample ($n < 30$), such graphs are not very helpful, so instead we use a different method.

Definitions

1. normal probability plot

2. normal score

If the sample data was taken from a normal random variable, then this plot should be approximately linear
Example 1 (#12 in your book) section 7.4
A random sample of weekly work logs at an automobile repair station was obtained and the average number of customers per day was recorded

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a) Let's only consider the first 5 and use a normal probability plot to assess whether the sample data could have come from a population that is normally distributed.

<table>
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<tr>
<th>$ith$</th>
<th>Actual Value in order</th>
<th>Area to the left of $ith$ value</th>
<th>Corresponding Z score</th>
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The area to the left of the 1st value is $f_i = \frac{i - 0.325}{n + 0.25}$

b) Plot the actual value against the corresponding zscores. (actual, zscores)

c) Use your calculator to plot.
## 7.5 - The Normal Approximation to the Binomial Distribution

**Definition to know:**

- **Correction for continuity** *(continuity correction)* is a correction employed when a continuous distribution is used to approximate a discrete distribution.
- Summary of the Normal Approximation to the Binomial Distribution

For all cases \( \mu = n \cdot p \), \( \sigma = \sqrt{npq} \) where \( np \geq 5 \) and \( nq \geq 5 \)

### Task 1

Suppose we have a Binomial Distribution with the following information: \( n=80 \) and \( p=0.4 \)

<table>
<thead>
<tr>
<th>Find the probability.</th>
<th>Using the normal distribution, estimate the probability</th>
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<tr>
<td>( P(x \leq 30) )</td>
<td>( P(x \leq 30) )</td>
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<td>( P(x &lt; 30) )</td>
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Example 1
Two out of five adult smokers acquired the habit by age 14.
If 400 smokers are randomly selected,
   a) find the probability that 170 or more acquired the habit by age 14

   b) using the normal distribution approximate the probability that 170 or more acquired the habit by age 14
Example 2
The percentage of female Americans 25 years old and older who have completed 4 years of college or more is 23.6%. In a random sample of 180, American women who are at least 25, what is the probability that more than 50 have completed 4 years of college or more?

Example 3
A magazine reported that 6% of American drivers read the newspaper while driving. If 300 drivers are selected at random, find the probability that exactly 25 say they read the newspaper while driving.