Activity 23 Factoring using the BOX
Math 40

Team Name (optional):
Your Name:
Partner(s): 1.
(2.)

Task 1: **Factoring out the greatest common factor**

Mini Lecture: Factoring polynomials is our focus now. Factoring can be thought of as "undoing multiplication". Unfortunately, no magic wand or algorithm exists for factoring polynomials. However, one method that some students find particularly useful is the box method.

Let's first examine the box method for the easiest case, factoring a monomial from a polynomial.

Recall how we could use the box method to multiply: Suppose we want to expand $2x(x+4)$. We can represent the resulting product as the area of a rectangle of length $x + 4$ and width $2x$.

	x	$+4$
$2x$	$2x^2$	$+8x$

We get the product $2x^2 + 8x$.

Now, let's work backwards to FACTOR. We begin by placing the terms of the polynomial inside the cells of the box.

$2x^2$	$+8x$

We then determine the greatest common factor of $2x^2$ and $8x$. Since both terms have a factor of 2 and a factor of x in common, the largest common factor is $2x$. Place this common factor outside and to the left of the cells.

$2x$	$2x^2$	$+8x$

Now, since $2x^2 = (2x)(x)$, we place the x above the $2x^2$. Also, since $8x = (2x)(4)$, we place the 4 above the $8x$.

	x	
$2x$	$2x^2$	$+8x$

	x	4
$2x$	$2x^2$	$+8x$

Thus, $2x^2 + 8x = 2x(x + 4)$.

Factor the following by using boxes.

1. $7x - 14$

2. $4x^2 + 6x$

3. $4y - 2$

4. $8m^2 - 18$

5. $5z^2 - 15z - 9$

Task 2: **Factoring by grouping**.

Mini Lecture: Often, the GCF is relatively easy to determine, so using the boxes seems unncessary. However, the boxes are incredibly helpful when we get to factoring other polynomials. Let's look at a few cases in which there is no common factor. The cases will involve polynomials with two terms (binomials), three terms (trinomials) and four terms.

In this example, we will factor a polynomial with four terms. For a polynomial with four terms we can factor by grouping.

Example: $3x^3 + 15x^2 - 2x - 10$

Each term can be placed in a corresponding cell of the box. Notice the degree of the term descends as you move from left to right and from top to bottom.

$3x^3$	$15x^2$
$-2x$	-10

Now we continue by factoring out the greatest common factor (GCF) of the terms in the first row (or the first column):

$3x^2$	$3x^3$	$15x^2$
	$-2x$	-10

Then we ask what times $3x^2$ will be $3x^3$ which gives the term above the first column, and what times $3x^2$ will be $15x^2$ which gives the term above the second column.

	x	$+ 5$
$3x^2$	$3x^3$	$15x^2$
	$-2x$	-10

Then fill in the term left of the bottom row in a similar manner.

	x	$+ 5$
$3x^2$	$3x^3$	$15x^2$
-2	$-2x$	-10

This the end result is $3x^3 + 15x^2 - 2x - 10 = (3x^2 - 2)(x + 5)$

Using boxes as done above, factor these polynomials having four terms:

1. $28x^3 - 35x^2 + 8x - 10$

2. $6x^3 - 10x^2 - 21x + 35$

3. $15x^3 + 20x^2 - 6x - 8$

4. $50x^3 + 125x^2 - 8x - 20$

Task 3: **Factoring Trinomials**.

Mini Lecture: Now, let's move to an example of a trinomial. First, let's review multiplying binomials:

Suppose we begin with a quadratic polynomial in factored form $(2x+3)(x+4)$. We can represent the product as the area of a rectangle of length $2x + 3$ and width $x + 4$.

	$2x$	$+3$
x	$2x^2$	$+3x$
$+4$	$+8x$	$+12$

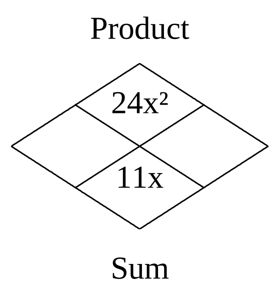
By combining the two linear terms, we get the product $2x^2 + 11x + 12$.

For FACTORING, we want to work backwards. We begin by placing the quadratic term $2x^2$ in the upper left cell and the constant term 12 in lower right cell.

$2x^2$	
	$+ 12$

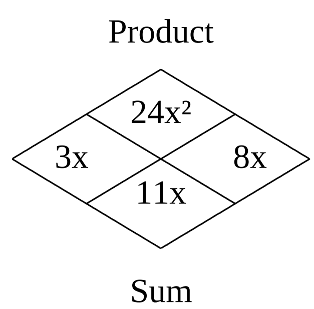
Next, we want to find the two terms that belong in the upper right and lower left. To do this we will use another schematic, the DIAMOND. In the top of the diamond we place the *product* of the first term and the last, $(2x^2)(+12) = 24x^2$. In the bottom we place the middle term, $11x$.

$2x^2$	?
?	$+ 12$



We are looking for the two terms whose product is $24x^2$ and whose sum is $11x$. You can start the search by looking at possible factors of $24x^2$. These two terms are placed in the sides of the diamond and also give us the terms that belong in the box.

$2x^2$	$+3x$
$+8x$	$+ 12$



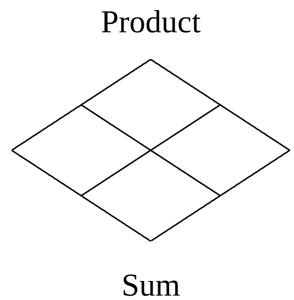
Now you are free to factor each row and column to arrive at the proper factorization.

	$2x$	$+3$
x	$2x^2$	$+3x$
4	$+8x$	$+12$

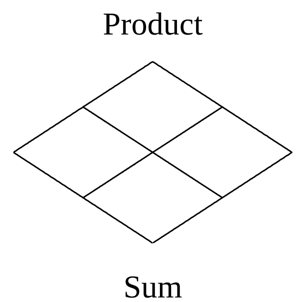
This will give the factored result. $2x^2 + 11x + 12 = (2x+3)(x+4)$

Factor the following quadratic trinomials using the box:

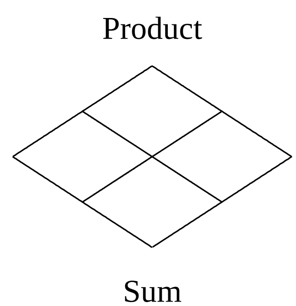
1. $5x^2 - 13x + 6$



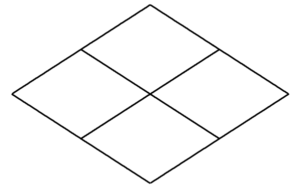
2. $x^2 + 8x + 16$



3. $3x^2 + 17x + 10$



Product

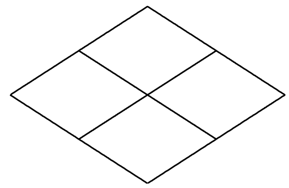


Sum

4. $x^2 - 12x + 32$

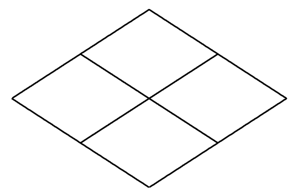
5. $5x^2 + 8x - 4$

Product



Sum

Product

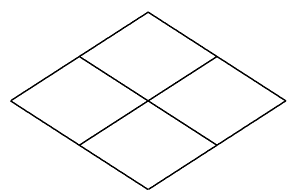


Sum

6. $x^2 - 3x - 18$

7. $7x^2 + 25x - 12$

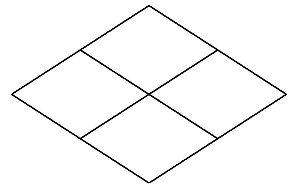
Product



Sum

8. $25x^2 - 40x + 16$

Product



Sum

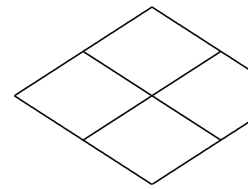
Task 4: **Factoring the difference of Squares**.

Now, let's consider factoring binomials. The only binomials that we factor in Math 40 are binomials whose terms have a common factor (as in task 1) and binomials which are the difference of squares. However, factoring the difference of squares can be done by adding a middle term and then using the trinomial method from Task 2. For example, $x^2 - 16 = x^2 + 0x - 16$.

Factor the following using the box method:

1. $x^2 - 16$ (which is equivalent to $x^2 + 0x - 16$)

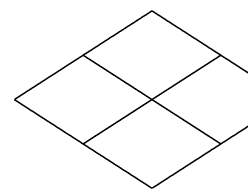
Product



Sum

2. $x^2 - 169$ (which is equivalent to $x^2 + 0x - 169$)

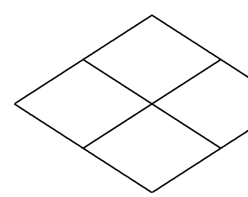
Product



Sum

3. $25x^2 - 16$ (which is equivalent to $25x^2 + 0x - 16$)

Product



Sum

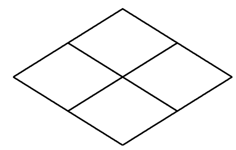
Task 5: **Simple Factoring First**, **then the Box**

The last thing you must recognize is that when you are asked to factor, it is expected that you factor completely. This means that you may use more than one of the methods described above. Consider Task 2 question 3. It is likely that when you factored the polynomial with four terms, you ended up with just two binomials. However, if you look carefully, you should see that one of the binomials is the difference of squares and can be factored further. Thus, the correct answer to should be a product of THREE binomials.

Factor the following completely.

1. $8x^2 - 20x + 12$ (Hint: First factor the GCF)

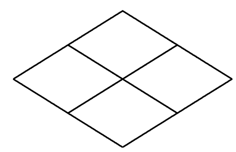
Product



Sum

2. $15x^4 + 12x^3 - 3x^2$ (Hint: First factor the GCF)

Product



Sum

3. $9x^3 - 12x^2 + 3x$

4. $16x^4 - 8x^3 - 8x^2$

5. $50x^3 + 125x^2 - 8x - 20$