Chapter 3 Math 43

Section 3.1/3.2
Exponents (Positive and Negative) and Scientific Notation

Simplifying Exponential Expressions

- The expression, $\mathbf{4^3}$ is an **exponential expression**.
    - **4 is the base** of this expression
    - **3 is the exponent**.

An expression of the form b^n is an ***exponential expression***, where ***b = base***, and ***n=exponent***.

So in general, $\mathbf{b^n}$ means to **multiply n b's** together $b^n = b \times b \times b \times b \times \ldots\ldots \times b$ If we could count all the b's on the right hand side of the equation, how many b's are being multiplied here?

TASK 1

1. Evaluate the following exponential expressions. Don't forget to follow the order of operations where necessary.

a) $(3 + 4)^2$ **b)** $3^2 + 4^2$

c) Does $(3 + 4)^2 = 3^2 + 4^2$? Why or Why not?

2. Using #1 above as an example, are the following statements true?

$(a + b)^2 = a^2 + b^2$ $(a - b)^2 = a^2 - b^2$

TASK 2: Properties of Exponents

Products of Powers

1. Review the following example in the table below.

$2^5 \cdot 2^3$	= $(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$
	= $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ (Multiplying eight 2's)
	= 2^8
$2^5 \cdot 2^3$	**=** 2^8

2. Use this pattern to simplify the following:

1) $4y^2 \cdot y^6$

b) $(6u^2v)(3uv^2)$

---

c) $(x^7)(x^9)$

d) $(st)^5(s^2t)^4$

---

3. **Rule for Exponents** $(a^n)(a^m) =$ ________________

TASK 3:

Quotient of Powers

1. Review the following example in the table below.

$\frac{2^5}{2^3} \quad = \quad \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$
$= \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = \frac{2 \cdot 2}{1} = 2 \cdot 2 =$ (two 2's)
$= \quad 2^2$

$\left(\frac{2}{x}\right)^3 \quad = \quad \left(\frac{2}{x}\right)\left(\frac{2}{x}\right)\left(\frac{2}{x}\right) = \frac{2 \cdot 2 \cdot 2}{x \cdot x \cdot x}$
$= \quad \frac{2^3}{x^3}$

2. Use this pattern to simplify the following:

a) $\frac{y^8}{y^3}$

b) $\frac{4^5 x^5}{4x^3}$

---

c) $\left(\frac{3x}{y^2}\right)^3$

d) $\frac{-21v^3}{12u^2 v}$

3. Rule for Exponents $\left(\frac{a^n}{a^m}\right) =$ ____________________

4. Rule for Exponents $\left(\frac{a}{b}\right)^n =$ ____________________

TASK 4:

1. Review the following examples below.

$\left(2^4\right)^3$ = $\left(2^4\right)\left(2^4\right)\left(2^4\right)$
= $(2\cdot2\cdot2\cdot2)(2\cdot2\cdot2\cdot2)(2\cdot2\cdot2\cdot2)$ = $(2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2)$ = twelve 2's
= 2^{12}

$(2xy)^3$ = $(2xy)(2xy)(2xy)$
= $2\cdot2\cdot2\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y$
= $2^3x^3y^3$

2. Use this pattern to simplify the following:

   a) $\left(a^2b\right)^3$

---

   b) $\left(-3y^2z\right)^2\left(2yz^2\right)^3$

---

3. Rule for Exponents $\left(a^n\right)^m =$ ________________

4. Rule for Exponents $(ab)^n =$ ____________________________

TASK 5:

Using the definition of exponents we can simplify $\frac{x^5}{x^7}$ by expanding the numerator and denominator.

$$\frac{x^5}{x^7} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x} =$$

Using the rule of exponents on page 3 of these notes, we can simplify $\frac{x^5}{x^7}$.

$$\frac{x^5}{x^7} = x^{5-7} =$$

Both are correct, and so both must be equal to each other.

Simplify the following expressions using BOTH the definition and Rule of Exponents.

a) $\frac{y^3}{y^8}$

---

b) $\frac{3x^4}{9x^9}$

---

5. Rule for Exponents $\left(a^{-n}\right) =$ ________________

TASK 5:

Using the definition of exponents we can simplify $\frac{x^5}{x^5}$ by expanding the numerator and denominator.

$$\frac{x^5}{x^5} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} =$$

Using the rule of exponents on page 3 of these notes, we can simplify $\frac{x^5}{x^5}$.

$$\frac{x^5}{x^5} = x^{5-5} =$$

Both are correct, and so both must be equal to each other.

Simplify the following expressions using BOTH the definition and Rule of Exponents.

a) $\frac{x^3}{x^3}$

---

b) $\frac{y^{-2}}{y^{-2}}$

6. Rule for Exponents $\left(a^0\right) =$ ________________

Practice Problems

a) $\left(-2v^2\right)\left(3v\right)\left(5v^5\right)$

---

b) $\dfrac{14c^4d^5}{7c^3d}$

---

c) $\left(r^2s^4\right)^3$

---

d) $\left(\dfrac{xy^2}{x^3y}\right)^4$

---

e) $\left(\dfrac{2w^2x^3}{3y^0}\right)^3$

---

f) $\dfrac{x^3y^{-2}}{s^{-4}t}$

g) $\left(\frac{-2a^{2}b^{-3}}{a^{-4}b^{-5}}\right)^{2}$

---

h) $\frac{3^{-4}n^{-3}}{-5p^{-2}}$

---

i) 4^{-3}

---

j) -2^{-5}

---

k) -2^{5}

---

l) 2^{-3}

---

m) $\frac{\left(a^{4}b^{-7}\right)^{-5}}{\left(5a^{2}b^{-1}\right)^{-2}}$

Section 3.3 and 3.4

Adding and Subtracting Polynomials

Definitions

a) Term

---

b) Coefficient

---

c) Constant term (constant)

---

d) Polynomial

---

e) Monomial, binomial, trinomial

---

f) Degree of a term

---

g) Degree of polynomial

---

h) Like terms

Example #1
Consider the polynomial

$$3x - 4x^2 - \frac{1}{3}$$

a) Identify all terms with their corresponding degrees

b) What is the degree of this polynomial?

c) How many terms does it have? Is it a monomial, binomial, or trinomial?

d) Is it in standard form? Why or why not?

e) If not in standard form, rewrite in standard form.

Example #2

Add or subtract the following and write your final answer in standard form.

a) $(3x^5 + 5x^3 - 5x^4 + 3x + 5) + (2x^5 + x^4 - 3x^2 + 3x - 4)$

---

b) $(x^5 - 4x^3 + x + 9) - (2x^4 - 3x^3 - 3)$

---

c) $(x^3 - 0.4x^2 - 12) - (x^5 - 0.3x^3 + 0.4x^2 - 9)$

---

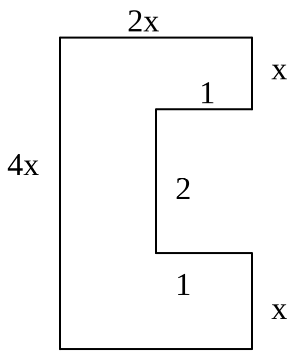
d) $\left(x^4 + \frac{2}{3}x + 5\right) + \left(4x^4 + 5x^2 + \frac{1}{3}x\right)$

---

e) $(5x^3 - x + 6) - (-9x^3 + 4x^2 - 6) + (3x - 5)$

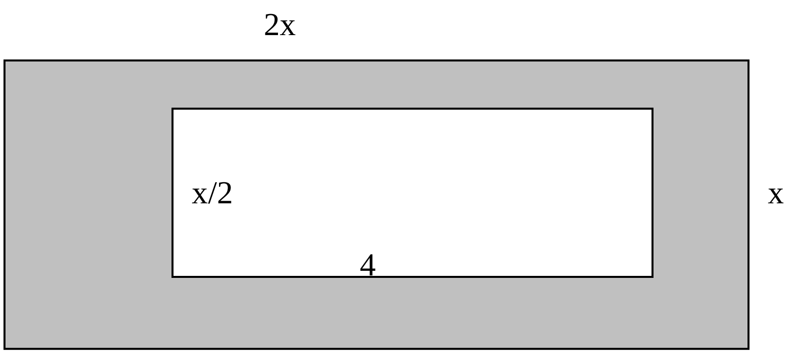
Example #3

Find an expression for the perimeter of the figure.



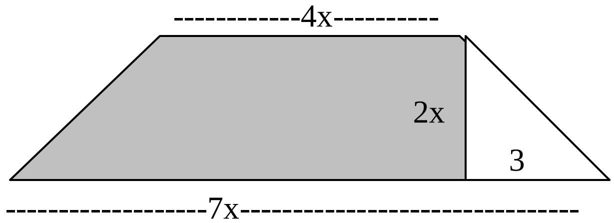
Example #4

Find an expression for the area of the figure.



Example #5

Find an expression for the area of the figure.



Section 3.5/3.6
Multiplying Polynomials

Multiplying Monomials

$$(3xy^2)(5x^4y^4z)$$

Multiplying Monomials with Binomials and Trinomials

a) $-6x(4x-4)$

b) $-7w^2y(6w^2+2wy-8y^2)$

Multiplying Binomials with Binomials - Using Boxes!!!

$(8x-5)(-x+1)$

Multiplying Polynomials of Any Number of Terms

$(x^2 + 2xy - y^2)(3x^2 - xy - 2y^2 + 3y)$

Multiplying Three or More Polynomials

a) $3x^2(x^2 + 3)(x - 4)$

b) $(3x - 3)(x^2 - 1)(4 - 5x^3)$

Squaring Binomials

$(3x-5)^2$

$(a+b)^2$

$\left(x-\frac{2}{5}\right)\left(x+\frac{1}{5}\right)$

$\left(\frac{x}{2}-4\right)^2$

$(3x+2)^2$

Section 3.7 - Dividing polynomials

- Case 1 – Dividing by a MONOMIAL
  If your divisor (the polynomial in the bottom) is a MONOMIAL … you may split up your problem

$$\frac{18x^4 - 24x^3 + 6x^2 - 12x}{6x} =$$

- **Case 2 – Dividing by a polynomial**
  If your divisor is NOT a monomial you have to perform LONG DIVISION.

$32\overline{)698}$	$\frac{6x^2+9x+8}{3x+2}$

a) $$\frac{y^2+8}{y+2}=$$

b) $$\frac{x^2+7x+10}{x+5}=$$

c) $$\frac{27x^3-8}{3x+2}=$$