2.7 Modeling with functions
1. define a problem.
2. find a function to model the problem.
3. find the extreme(s) of the problem.

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#2. A poster is 10 inches longer than it is wide. Find a function that models its area in terms of its width w.
Analysis:
Area of the poster
shape of the poster: rectangle: length \((w + 10)\), width \(w\)
Area = length \(\cdot\) width
\(= (w + 10) \cdot w\)
Solution: Since the width of the poster is \(w\), the length is \(w + 10\).
Then the area of the poster is
Area \(A(w) = (w + 10) \cdot w\)
this is the area of the poster.

#4. The height of a cylinder is four times its radius. Find a function that models the volume of the cylinder in terms of its radius \(r\).
Solution: If the radius is \(r\), the height of the cylinder is \(4r\).
Then the volume of the cylinder is
\[V(r) = \pi r^2 \cdot 4r = 4\pi r^3\]

#24. A rancher with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle.
(a) Find a function that models the total area of the four pens.
(b) Find the largest possible total area of the four pens.
Solution: Let the width of the rectangle, which is the same as the length of a pen) be \(x\). From the picture, the length of the rectangle, say \(L = \frac{750-5x}{2}\). Since there are 4 pens with equal width, the width of one pen is \(L/4 = \frac{750-5x}{8}\).
The area of one pen is \(x \cdot \frac{750-5x}{8}\), the area of 4 pens, then
\[A(x) = 4x \cdot \frac{750-5x}{8} = x \cdot \frac{750-5x}{2} = \frac{(750-5x)x}{2} = \frac{750x-5x^2}{2}\]
\[A(x) = -\frac{5}{2}x^2 + 375x\]
(b) In this parabola function, \(a = -\frac{5}{2} < 0\), the function exists maximum \(k\) for a value of \(h\).
Goal \(\Rightarrow\) \(a(x-h)^2 + k\)
\[A(x) = -\frac{5}{2}x^2 + 375x\]
\[= -\frac{5}{2}(x^2 - 150x)\]
\[= -\frac{5}{2}(x^2 - 150x + 75^2 - 75^2)\]
\[= -\frac{5}{2}[(x^2 - 150x + 75^2) - 75^2]\]
\[
\begin{align*}
&= -\frac{5}{2} \left[ (x - 75)^2 - 75^2 \right] \\
&= -\frac{5}{2} (x - 75)^2 - \left( -\frac{5}{2} \right) 75^2 \\
&= -\frac{5}{2} (x - 75)^2 + \frac{5}{2} \cdot 75^2 \\
&= -\frac{5}{2} (x - 75)^2 + \frac{5 \cdot 75^2}{2}
\end{align*}
\]

If \( x = h = 75 \), \( A(75) \) reaches maximum, \( k = \frac{5 \cdot 75^2}{2} \)

The largest possible area of the 4 pens is \( \frac{5 \cdot 75^2}{2} \) ft\(^2\)