Problem 1 (18 points) Evaluate function: \( f(x) = \frac{x-1}{x+1} \) at the indicated values:

\[
\begin{align*}
f(2) &= \frac{2-1}{2+1} = \frac{1}{3} \\
f(-2) &= \frac{-2-1}{-2+1} = -3 \\
f\left(\frac{1}{2}\right) &= \frac{\frac{1}{2}-1}{\frac{1}{2}+1} = \frac{-\frac{1}{2}}{\frac{3}{2}} = -\frac{1}{3} \\
f(a) &= \frac{a-1}{a+1} \\
f(1) &= \frac{1-1}{1+1} = \frac{0}{2} = 0 \\
f(-1) &= \frac{-1-1}{-1+1} = \frac{-2}{0} = \text{Undefined}
\end{align*}
\]

Please fill your answer in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = \frac{x-1}{x+1} ) answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>-( \frac{1}{3} )</td>
</tr>
<tr>
<td>( a )</td>
<td>( \frac{a-1}{a+1} )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-( \infty ) (Undefined)</td>
</tr>
</tbody>
</table>

Problem 2 (12 points) Find the domain of the following functions:

(a) \( f(x) = \frac{x}{x^2-1} \)

\( x^2 - 1 \neq 0 \)
\( (x+1)(x-1) \neq 0 \)
\( x \neq -1 \) and \( x \neq 1 \)

Domain: \( \{x | x \neq \pm 1\} \)

Note: for (b), more detailed work shown here:

Problem 3 (10 points) Given \( f(x) = x^2 - 1 \), Find \( \frac{f(1+h) - f(1)}{h} \).

\[
\begin{align*}
f(1+h) &= (1+h)^2 - 1 = 1 + 2h + h^2 - 1 = 2h + h^2 \\
f(1) &= 1^2 - 1 = 0 \\
\frac{f(1+h) - f(1)}{h} &= \frac{2h + h^2 - 0}{h} = \frac{(2+h)h}{h} = \frac{(2+h)}{1} = 2 + h
\end{align*}
\]

Problem 4 (10 points) Solve the following equation, be sure to check your answers:

\[
\frac{x}{x^2} + \frac{1}{x^2} = \frac{8}{x^2-4}
\]

Solution #1.

\[
\begin{align*}
\frac{x(x+2)}{(x-2)(x+2)} + \frac{1(x-2)}{(x+2)(x-2)} &= \frac{8}{(x-2)(x+2)} \\
\frac{x(x+2) + (x-2)}{(x-2)(x+2)} &= \frac{8}{(x-2)(x+2)} \\
\frac{2x^2}{(x-2)(x+2)} &= \frac{8}{(x-2)(x+2)} \\
2x^2 &= 8 \\
x^2 &= 4 \\
x &= \pm 2
\end{align*}
\]
\[
\frac{x(x+2)+(x-2)-8}{(x-2)(x+2)} = 0
\]
\[
\frac{x+2}{x+3x-10} = 0
\]
\[
\frac{x+5}{x+2} = 0
\]
\[x + 5 = 0 \Rightarrow x = -5\]

Solution #2. \( \text{LCD} = (x-2)(x+2) \)
\[
\frac{1}{x+2} \cdot (x-2)(x+2) + \frac{x}{x+2} (x-2)(x+2) = \frac{8}{(x-2)(x+2)} (x-2)(x+2)
\]
\[x (x+2) + (x-2) = 8\]
\[x^2 + 3x - 10 = 0\]
\[(x + 5)(x - 2) = 0\]
\[x = -5, x = 2\]
Check: \(x = 2\) is not valid for the original equation.
Solution \(x = -5\)

1. Problem 5. (10 points) Because of an anticipated heavy rainstorm, the water level in a reservoir must be lowered by 1 ft. Opening spillway number \(A\) does this task in 4 hours, whereas opening the smaller spillway number \(B\) does this task in 6 hours. How long will it take to lower the water level by 1 ft if both spillways are opened?

<table>
<thead>
<tr>
<th></th>
<th>Amt time needed</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spillway A</td>
<td>4</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>Spillway B</td>
<td>6</td>
<td>(\frac{1}{6})</td>
</tr>
</tbody>
</table>

\[t: \text{number of hours it takes for both to do the task.}\]
\[\frac{1}{t} = \frac{1}{4} + \frac{1}{6}\]

\[\text{solve for } t.\]
\[\text{LCD} = 12t\]
\[
\frac{1}{t} \cdot 12t = \frac{1}{4} \cdot 12t + \frac{1}{6} \cdot 12t
\]
\[12 = 3t + 2t\]
\[5t = 12 \Rightarrow t = \frac{12}{5} = 2 \frac{2}{5} \text{ hours.}\]

It will take \(2 \frac{2}{5}\) hours for both spillways to lower the water level by 1 ft.

Problem 6. (10 points) Rationalize as required:
(a) \(\frac{1}{\sqrt{x} - 1}\) (Rationalize the denominator)
\[
\frac{1}{\sqrt{x} - 1} = \frac{1(\sqrt{x} + 1)}{(\sqrt{x} - 1)(\sqrt{x} + 1)} = \frac{\sqrt{x} + 1}{(\sqrt{x})^2 - 1} = \frac{\sqrt{x} + 1}{x - 1}
\]
(b) \(\frac{\sqrt{x+h} - \sqrt{x}}{h}\) (Rationalize the numerator)
\[
\frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} = \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}
\]

Problem 7. (20 points) Write the resulting function after the indicated transformations:
<table>
<thead>
<tr>
<th>Function</th>
<th>Type of transformation</th>
<th>Resulting function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^2$</td>
<td>shift to the left by 2 units.</td>
<td>$f(x + 2) = (x + 2)^2$</td>
</tr>
<tr>
<td>$g(x) = 2x^3 + x$</td>
<td>shift up by 1 units.</td>
<td>$g(x) + 1 = 2x^3 + x + 1$</td>
</tr>
<tr>
<td>$h(x) = \sqrt{x}$</td>
<td>Reflect with respect to $x$–axis</td>
<td>$-h(x) = -\sqrt{x}$</td>
</tr>
<tr>
<td>$k(x) = \frac{1}{x}$</td>
<td>reflect with respect to $y$–axis</td>
<td>$k(-x) = \frac{1}{-x} = -\frac{1}{x}$</td>
</tr>
</tbody>
</table>

Problem 8. (10 points) Determine whether the given functions are even, odd, or neither.

<table>
<thead>
<tr>
<th>Function</th>
<th>even, odd or neither?</th>
<th>Your answer here this column</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^3 + x$</td>
<td>odd</td>
<td></td>
</tr>
<tr>
<td>$g(x) = x^2 - x$</td>
<td>neither</td>
<td></td>
</tr>
<tr>
<td>$f(-x) = (-x)^3 + (-x) = -(x^3 + x) = -f(x)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g(-x) = (-x)^2 - (-x) = x^2 + x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g(-x) \neq g(x)$ and $g(-x) \neq -g(x)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Work for extra credits

Problem 9. (10 points) Mercedes starts jogging 5 miles per hour. 30 minutes later, Karen starts jogging on the same route at 7 miles per hour. How long will it take Karen to catch Mercedes?

$t$: time it takes Karen to catch up with Mercedes.

<table>
<thead>
<tr>
<th>rate (mph)</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercedes</td>
<td>$t + \frac{1}{2}$</td>
</tr>
<tr>
<td>Karen</td>
<td>$t$</td>
</tr>
</tbody>
</table>

distance run by Mercedes = distance run by Karen

$5(t + \frac{1}{2}) = 7t$

$2t = \frac{5}{2} \Rightarrow t = \frac{5}{4} = 1\frac{1}{4}$ hours.

It takes Karen $1\frac{1}{4}$ hours to catch up with Mercedes.

Problem 10. (10 points) It takes Pat 12 hours to complete a task. After he had been working for 3 hours, his was joined by his brother, Mike, and together they finished the task in 5 hours. How long would it take Mike to do the job by himself?

<table>
<thead>
<tr>
<th>Time to finish alone</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat</td>
<td>$\frac{1}{12}$</td>
</tr>
<tr>
<td>Mike</td>
<td>$\frac{1}{t}$</td>
</tr>
</tbody>
</table>

first 3 hours, done by Pat, finished amount of work: $3 \cdot \frac{1}{12} = \frac{3}{12}$

Next 5 hours: $\frac{1}{12} \cdot 5 + \frac{1}{t} \cdot 5 = 1 - \frac{3}{12}$

$t = 15$ hours.

It takes Mike 15 hours to do the job by himself.