Goals: To explore linear inequalities with functions.

Task 1: (group work warm-up exercises – use your calculators if you want) Renting a Pick-up Truck: Jeff needs to move some lumber from his mountain villa to his city residence. The one-day cost functions for renting a pick-up truck from Budget or U-Haul are

\[ B(x) = 39.95 + 0.19x \]

\[ U(x) = 19.95 + 0.48x, \]

where \( x \) represents the number of miles driven. Before looking at a graph of these two linear functions, answer the following questions:

1. When \( x = 10 \) miles, which is more expensive, the pick-up truck from Budget or the pick-up truck from U-Haul? Answer this question by first evaluating both \( B(10) \) and \( U(10) \). Then explain your answer by completing the following sentence: "If Jeff needs to drive 10 miles, then...."

2. Answer Question 1 in the following form (by filling in the box with <, >, or =):

   When \( x = 10 \) miles, \( B(10) \underline{=} U(10) \).

3. When \( x = 100 \) miles, which is more expensive, the pick-up truck from Budget or the pick-up truck from U-Haul? Answer this question by first evaluating both \( B(100) \) and \( U(100) \). Then explain your answer by completing the following sentence: "If Jeff needs to drive 100 miles, then...."

4. Answer Question 3 in the following form (by filling in the box with <, >, or =):

   When \( x = 100 \) miles, \( B(100) \underline{=} U(100) \).

5. Your answers in Questions 2 and 4 should be opposites. That means that, up to some particular \( x \)-value (let’s call it \( x_0 \), which we read as "ex-sub-zero"), we have \( B(x) \underline{=} U(x) \) (fill in the box with < or >). And after \( x_0 \), the inequality switches, and \( B(x) \underline{=} U(x) \) (fill in the box with < or >). At \( x_0 \), the costs of renting a pick-up truck at Budget and at U-Haul are equal, that is to say, \( B(x_0) = U(x_0) \).
6. Find \( x_0 \). (You know how to do this. Set the expressions for \( B(x) \) and \( U(x) \) equal to each other and solve for \( x \). Use your calculator to get your final answer and round it to two decimal places.)

**Task 2: (Mini-Lecture)** Now let’s look at the graph of these two functions. Label each line correctly (\( C = B(x) \) or \( C = U(x) \)). It is clear that to the left of \( x_0 \), the \( B(x) \)-values are greater than the \( U(x) \)-values. To the right of \( x_0 \), the opposite holds.

An Abstract Example: Next, let’s look at a more abstract example. Here we have two linear functions \( f(x) = 6 - 3x \) and \( g(x) = 2x - 1 \). Below are the graphs of the two functions, on the same grid.

1. In the graph below, identify which line is \( y = f(x) \) and which line is \( y = g(x) \). Label them clearly.

2. Find the point of intersection. The ordered pair of this point can be described as \( (x_0, f(x_0)) \) or \( (x_0, g(x_0)) \). After all, it is the point of intersection. Label the value of \( x_0 \) on the graph.

3. Fill in the blanks in the picture below with the proper inequality sign (\(< \) or \(>\)).
Task 4: (Group Work) An Even Easier Real-Life Example: Let’s go back to Jeff’s pick-up truck rental. Suppose U-Haul offers a special rental deal where the flat rate to rent the pick-up for the day is $55.55 with no additional charge per mile. Budget keeps the same rental deal as before. In other words, now the two functions are

\[ B(x) = 39.95 + 0.19x \]
\[ U(x) = 55.55 \]

The graphs of these two functions is given in the figure below.

1. In the figure below, identify which line is \( C = B(x) \) and which line is \( C = U(x) \). Label them clearly.

2. Find the point of intersection. Label the value of \( x_0 \) on the graph.

3. Fill in the blanks in the picture below with the proper inequality sign (< or >).

\[ C, \text{ cost in dollars} \]
\[ B(x) \_ \_ 55.55 \]
\[ x_0 \]
\[ B(x) \_ \_ 55.55 \]

Task 5: (Mini-Lecture) An Easy Abstract Example: Let’s go back to the textbook for the last two examples. The problem asks you to find the solution set for the inequality

\[ x - 3 < 8. \]

We know how to find the solution set for this inequality using algebra. Simply add 3 to both sides of the inequality. We get \( x < 11 \).

We’re going to look at this problem from a function point of view. We are going to let \( f(x) = x - 3 \) and \( g(x) = 8 \). We know the point of intersection of these two graphs is the point \((11,8)\).

In the graphs of these two functions, we know that the \( x \)-value for the point of intersection (which we’ve been calling \( x_0 \) in our previous examples) is equal to 11. So in the picture on the next page, we see that the piece of the graph of \( f(x) = x - 3 \) that lies to the left of the vertical line \( x_0 = 11 \) also lies below the horizontal line \( y = 8 \). In other words, if \( x < 11 \), then \( x - 3 < 8 \). The thick portion of the \( x \)-axis (with the arrow pointing to the left) is the solution set for our original inequality \( x - 3 < 8 \).
Task 6: (Group Work) Now, let’s look at another example. The inequality is \(-x + 2 \geq 5\). Solve this inequality algebraically:

Again, we want to interpret this inequality using functions \(f(x) = -x + 2\) and \(g(x) = 5\). Below are the graphs of these two functions.

1. On the graph above, draw the vertical line \(x = -3\).

2. Look at the graph of \(f(x)\). The graph of \(y = f(x)\) is greater than the graph of \(y = 5\) on one side of the vertical line \(x = -3\). Decide which side it is.

3. Highlight or thicken the portion of the \(x\)-axis that is on the same side of the vertical line \(x = -3\) that you chose question 2. Place a solid dot at \(x = -3\) on the \(x\)-axis. This is your number line solution set for the inequality \(-x + 2 \geq 5\).