Introduction: A rational expression is a fraction with polynomials as the numerator and denominator. In this activity you will learn how to simplify a rational expression. If you follow the procedure of this activity you will simplify correctly every time.

First, let’s quickly look at how NOT to simplify. Here is a typical error of many students who do not understand what it means to simplify.

\[
\frac{2x^2 + 10}{2x^2 + 2} = \frac{10}{2} = 5 \quad \text{by "canceling" the } 2x^2 \text{ in the top with the } 2x^2 \text{ in the bottom.}
\]

One way to see this is incorrect is to substitute a number in place of \( x \) and see that you do not get the same result. For example let \( x = 1 \), then

\[
\frac{2(1)^2 + 10}{2(1)^2 + 2} = \frac{12}{4} = 3
\]

Now, let’s look at the correct, fool-proof way to simplify. We will use lots of parentheses as they are free, they can be thrown away when no longer needed and they don’t pollute the environment! Here are the steps to reducing a rational expression.

\[
\frac{16x^3}{12x^{10}}
\]

1. Give the numerator and the denominator each its own set of parentheses.

\[
= \frac{(16x^3)}{(12x^{10})}
\]

2. Factor the numerator and the denominator and give each factor (including monomial and constant factors!) its own set of parentheses.

\[
= \frac{(4)(4)(x^3)}{(3)(4)(x^3)(x^7)}
\]

3. Identify and then remove ("cancel") parentheses (factors) in the numerator and the denominator that are identical.

\[
= \frac{(4)}{(3)}
\]

4. (Optional) Remove any unnecessary parentheses.

\[
= \frac{4}{3x^7}
\]
Here are two (2) more examples.

\[
\frac{x^3 - 5x^2 + 6x}{x^3 - 2x^2 - 9x + 18} = \frac{(x^3 - 5x^2 + 6x)}{(x^3 - 2x^2 - 9x + 18)} = \frac{(x)(x^2 - 5x + 6)}{(x^2)(x - 2) - (9)(x - 2)} = \frac{(x)(x - 2)(x - 3)}{(x)(x - 2)(x - 3)} = \frac{x}{x + 3}
\]

\[
\frac{5 - x}{x - 5} = \frac{(5 - x)}{(x - 5)} = \frac{(-1)(-x + 5)}{(x - 5)} = \frac{(-1)(x - 5)}{(x - 5)} = -1
\]

**Task 1. Use the method described above to simplify the following expressions**

a) \( \frac{5x^8}{15x^3} \)

b) \( \frac{3y^2}{27y^5} \)

c) \( \frac{6x - 4}{9x^2 - 4} \)

d) \( \frac{12x^2 - 75}{6x^2 - 39x + 60} \)

e) \( \frac{7 - x}{x^2 - 49} \)
Task 2. Simplify the right side of the equation.
Verify your result by comparing either graphing calculator tables or graphs for the original function with that of the simplified form.

a) \( f(x) = \frac{3-x}{x-3} \)

b) \( f(x) = \frac{-2x+10}{4x-20} \)

c) \( f(x) = \frac{25-4x^2}{2x^2-21x+40} \)

Task 3. Try to explain the following.
A student tries to reduce the expression \( \frac{(x-2)(x-1)}{4(x-2)+5} \) as follows:

\[ \frac{(x-2)(x-1)}{4(x-2)+5} = \frac{x-1}{4+5} = \frac{x-1}{9} \]

Next, the student substitutes 3 for \( x \) in the original expression, with the result:

\[ \frac{(3-2)(3-1)}{4(3-2)+5} = \frac{(1)(2)}{4+5} = \frac{2}{9} \]

Now, the student substitutes 3 for \( x \) in the reduced expression, with the result:

\[ \frac{x-1}{9} = \frac{3-1}{9} = \frac{2}{9} \]

The student then concludes that the result \( \frac{x-1}{9} \) is correct.

Try to explain either why the student is correct or why the student is incorrect.