Activity 3.3

Trigonometric Functions of Real Numbers

Your Name: 
Partner(s): 1. 
(2.)

Objective: To extend the definition of the trigonometric functions of angles to functions of real numbers.

Warm-Up Under what two conditions is the length of an arc, \( t \), equal to the measure of the central angle, \( \theta \), that subtends it?

\[ t = \theta \quad \text{when} \]

1. 

2. 

Introduction Let \( t \) be any real number. We can represent the real number \( t \) as the length of a directed arc of a unit circle, starting at the point \((1,0)\), and ending at a point \((x,y)\). If the real number is positive, the arc is traversed counterclockwise. If the real number is negative, the arc is traversed clockwise. The number \( t \) can be associated with the arclength subtended by the radian measure of an angle in standard position. Notice that a positive rotation (counterclockwise) corresponds to a positive number \( t \).

Then

\[ \sin(t) = y \quad \cos(t) = x \]

Or we can also say that the coordinates of the point on the unit circle are given by

\[ (x,y) = (\cos(t), \sin(t)) \]
**Task 1.** The unit circle below is cut into eight equal arcs so that each arc has a length of $\frac{\pi}{4}$. Label the coordinates of each of the eight points on the circle. This is the same as you did in the previous activity.

![Unit Circle](image)

**Task 2.** The unit circle below is cut into 12 equal arcs so that each arc has a length of $\frac{\pi}{6}$. Label the coordinates of each of the 12 points on the circle.

![Unit Circle](image)

**Task 3.** Complete the following table using the above information to help you.

<table>
<thead>
<tr>
<th>$t$</th>
<th>sin($t$)</th>
<th>cos($t$)</th>
<th>tan($t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{6}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{4}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{3}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>