Objective: To model a compression sound wave with a sine function and interpret the transformations in terms of features of sound.

Introduction: Like all sound, music is created by compression waves traveling through the air. An impulse sound compresses the air causing an increase in pressure near the origin of the sound. The pressure is released by causing air farther away to compress reducing the pressure of the nearby air. The phenomenon is the same as a pebble hitting water causing ripples to emanate from the point where the pebble hit. The waves have depressions (corresponding to the high pressure and crests (corresponding to low pressure). A model is given by the following diagram.

On the wave graph above, the vertical axis represents air pressure and the horizontal axis is time. The amplitude of the wave corresponds to the loudness of the sound. The frequency, measured in cycles per second or herz (Hz) is the inverse of the period, the time from peak to peak. Frequencies detectable by the human ear range from 20 Hz to 20,000 Hz. In music the frequency is referred to as pitch.
Task 1. **Modeling Air Pressure with a Sine Function.**

Suppose a tuning fork is struck emitting a pure "A" note by vibrating at a frequency of 440 Hz. At a particular location the air pressure increases and decreases on a regular cycle as illustrated by the graph above. Since the air pressure is a periodic function of time, we can let \( f(t) = \sin(\omega t) \) represent the air pressure as a function of time in seconds.

a) What is the period of the function?

b) What is the value of \( \omega \)? This is called the angular frequency and is measured in radians per second.

c) Rewrite the function using the value of the angular frequency.

d) Sketch three cycles of this "A" note on the grid below.
Task 2.  More Accurate Models

What makes most sounds, including music, so rich are the overtones. If the string of a guitar vibrates at a fundamental frequency of 440 Hz, it also vibrates at many other frequencies as well, albeit with reduced loudness. These subtleties are called overtones or harmonics and give each instrument its characteristic sound or timbre.

For a single vibrating string the overtones occur at frequencies which are multiples of the fundamental frequency. So a string vibrating at 440 Hz will also vibrate at 880 Hz, 1320 Hz, etc. Mathematically this is modeled by combining sine waves of different frequencies and amplitudes.

A simple example of a combination wave, called a Fourier expansion, is given below:

\[ f(t) = 2\sin(t) - \sin(2t) + \frac{2}{3}\sin(3t) - \frac{1}{2}\sin(4t) \]

If we graph this together with \( y = 2\sin(t) \), representing the fundamental frequency, we can see how it dominates.

\[ f(t) = 2\sin(t) - \sin(2t) + \frac{2}{3}\sin(3t) - \frac{1}{2}\sin(4t) \]

Notice how the period of the fundamental wave and the period of the combination wave, the Fourier expansion, are the same \( 2\pi \).
1) What are the frequencies of each of the four terms?

2) What is the amplitude (loudness) of the term which has a frequency of \( \frac{1}{\pi} \)?

3) What is the amplitude (loudness) of the term which has a frequency of \( \frac{2}{\pi} \)?

Incredibly the human ear is equipped to pick out the variations in very complex waves with many terms representing the many overtones. Tiny hairs on a membrane in the ear vibrate at different frequencies and send signals to the brain telling it how loud each frequency is. The human ear does what is mathematically called Fourier analysis - in a fraction of a second!