1.1 Velocity and Distance

What is the relationship between velocity and distance? The answer is the key idea of calculus. Suppose we know the entire history of the distance traveled by a car. Can we determine its velocity? And vice versa suppose we know the history of the velocity of the car, can we determine its distance? To find the velocity from the distance is the subject of **differential calculus**. To find the distance from the velocity if the subject of **integral calculus**. Primarily in this semester we will be concerned with the former. If \( f \) represents the distance traveled and \( v \) the velocity. **Differentiation** takes us from \( f \) to \( v \). And **integration** takes us from \( v \) to \( f \). Let’s consider some special cases.

**Example 1**  
Constant velocity.

Suppose after traveling a distance of 50 km we continue at a constant velocity \( v = 100 \) km/h. Then \( f \) increases by 100 km each hour and continues to increase at this constant rate.

![Distance Graph](image1.png)

Notice that the **slope of the \( f \) graph is the velocity** \( v = 100 \). What is the equation for \( f \)?

\[
\frac{df}{dt} = v
\]

So the velocity is the slope of the distance graph. **To calculate the slope of \( f \) is differentiation**.

Now, how do we get the distance from the velocity graph? The answer is the area! What is the area under the \( v \) graph from \( t = 1.0 \) to \( t = 1.5 \)?

\[
\text{Area under } v \text{ from } 1 \text{ to } 1.5 =
\]

What is the difference in the \( f \) values at those times?

\[
\text{Difference in } f \text{ from } 1 \text{ to } 1.5 =
\]

Okay so we cannot get the distance traveled, only the difference in the distance traveled. **To calculate the area under \( v \) is integration**.

**Example 2.**  
Piecewise-constant velocity.

Suppose we now imagine the car traveling at constant velocity of \( v = 100 \) km/h and then returning at the same speed. Here are the graphs.

![Velocity Graph](image2.png)
Notice that the slope of the $f$ graph is the velocity $v = 100$ for $0 < t < 1.5$ and $v = -100$ for $1.5 < t < 3$. What is the equation for $f$?

$$f(t) = \begin{cases} 100t + 50 & \text{for } 0 < t < 1.5 \\ \end{cases}$$

Again the velocity is the slope of the distance graph. This is differentiation.

To get the distance from the velocity graph we have to allow negative area! What is the area under the $v$ graph from $t = 1.0$ to $t = 3$?

Area under $v$ from 1 to 3 =

What is the difference in the $f$ values at those times?

Difference in $f$ from 1 to 3 =

This is integration.

**Task 1.** Velocity from the Distance Graph

a) Use the given the distance graph above to draw the velocity graph.

b) What is the "net" area under $v$ from $t = 1$ to $t = 4$?

c) What is $f(4) - f(1)$?

d) Write a formula for $f$.

$$f(t) = \begin{cases} \end{cases}$$
e) State the domain and range of $f$.
\[ \text{dom } f = \]
\[ \text{ran } f = \]

f) Write a formula for $v$.
\[ v(t) = \]

\[ \text{dom } v = \]
\[ \text{ran } v = \]

(Here you need a union of intervals.)
(Here you need set notation, not interval notation.)

**Task 2. Distance from Velocity Graph**

![Velocity Graph]

a) Assuming that $f(0) = 0$, draw the graph of $f$, whose velocity is given above.

![Position Graph]

b) Write formulas for $v$ and $f$.

**Task 3. Linear Functions, $y(t) = vt + C$**

a) Write a formula for the linear function with $y(1) = 3$ and slope $\frac{1}{2}$.

b) Write a formula for the linear function with $y(t - 2) = y(t) + 6$ and $y(1) = 2$
a) Sketch the graph of $f(t) = t - 2 - |t - 2|$, for $0 \leq t \leq 5$. What is the slope and range?

b) Suppose $v = 2$ from $t = 0$ to $t = 4$. What constant velocity from $t = 4$ to $t = 6$ would make $f(6) = 20$ if $f(0) = 0$? Hint: Sketch a graph and use areas. Write a formula for $f$. 