Chapter 5. Decimals

Use the calculator.

5.1 An Introduction to the Decimals

1. Identify the place value of a digit in a decimal number
2. Write decimals in expanded form
3. Read decimals and write them in standard form
4. Compare decimal using inequality symbols
5. Graph decimals on a number line
6. Round decimals
7. Read tables and graphs involving decimals

5.1 An Introduction to the Decimals

1. Identify the place value of a digit in a decimal number

This is a review.

e.g.1 Consider the decimal number: 8,420.167953

a. What is the place value of the digit 5?

Ans. hundred thousandths

b. Which digit tells the the number of hundredths?   Ans. 6

5.1 An Introduction to the Decimals

2. Write decimals in expanded form

This is a review.

e.g.2 Write the decimal number 549.2136 in expanded form.

$$\text{Ans.} \quad 500 + 40 + 9 + \frac{2}{10} + \frac{1}{100} + \frac{3}{1,000} + \frac{6}{10,000}$$

5.1 An Introduction to the Decimals

3. Read decimals and write them in standard form

This is a review.

e.g.3 Write each decimal in words and then as a fraction or mixed numbers.

a. One gallon of milk weighs 8.6 lbs

Ans. eight and sixth

Ans. $8\dfrac{6}{10}$

b. One meter is about 39.3701 inches

Ans. thirty-nine and three thousand, seven hundred one ten-thousandths

Ans. $39\dfrac{3,701}{10,000}$

5.1 An Introduction to the Decimals

3. Read decimals and write them in standard form

Mostly review.

e.g.3 Write each decimal in words and then as a fraction or mixed numbers.

c. The smallest freshwater fish is the dwarf pygmy goby. Adult males weigh 0.00014 ounce.

Ans. fourteen hundred-thousandths

Ans. $\dfrac{14}{100,000}$

d. Ocean salt water freezes at –1.94° Celsius

Ans. negative one and ninety-four hundredths

Ans. $-1\dfrac{94}{100}$

5.1 An Introduction to the Decimals

3. Read decimals and write them in standard form

This is a review.

e.g.4 Write each number in standard form

a. Sixty-seven and thirty-eight hundredths   Ans. 67.38

b. Nineteen and twenty-five thousandths   Ans. 19.025

5.1 An Introduction to the Decimals

4. Compare decimals using inequality symbols

- Align decimal point.
- Compare digits, column by column, from left to right
- The first time when two digits are different
  - If decimals are positive, the decimal with greater digit is the greater of two decimals
  - If decimals are negative, the decimal with the smaller digit is the greater of two decimals

5.1 An Introduction to the Decimals

4. Compare decimals using inequality symbols

Mostly review.

e.g.5 Place an < or > symbols in the box to make a true statement:

a.   2.7446 $\boxed{>}$ 2.7439

b.   39.2 $\boxed{<}$ 39.203

c.   −10.45 $\boxed{<}$ −10.419
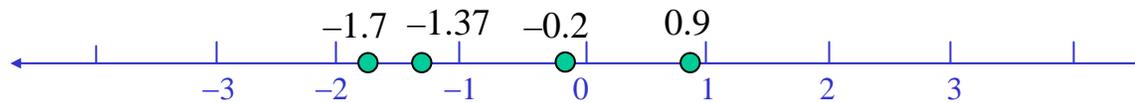
5.1 An Introduction to the Decimals

5. Graph decimals on a number line

Negative number on a number line.

e.g.6 Graph –1.37, 0.9, –1.7, and –0.2

5.1 An Introduction to the Decimals

6. Round decimals

Same rule for rounding negative numbers.

e.g.8 Round each decimal to the indicated place value:

a.　　–420.139 to the nearest tenth　　Ans.　–420.1

b.　　56.095 to the nearest hundredth　　Ans.　56.10

e.g.9 UTILITY BILLS A utility company calculates a homeowner's monthly natural gas bill by multiplying the unit cost of $0.96412 by the number of therms used that month. Found the unit cost to the nearest cent.　Ans.　$0.96

5.1 An Introduction to the Decimals

6. Round decimals

e.g.9 A plumber earned $76,350.87 dollars in on year.
Round his income to the nearest dollar.   Ans. $76,351
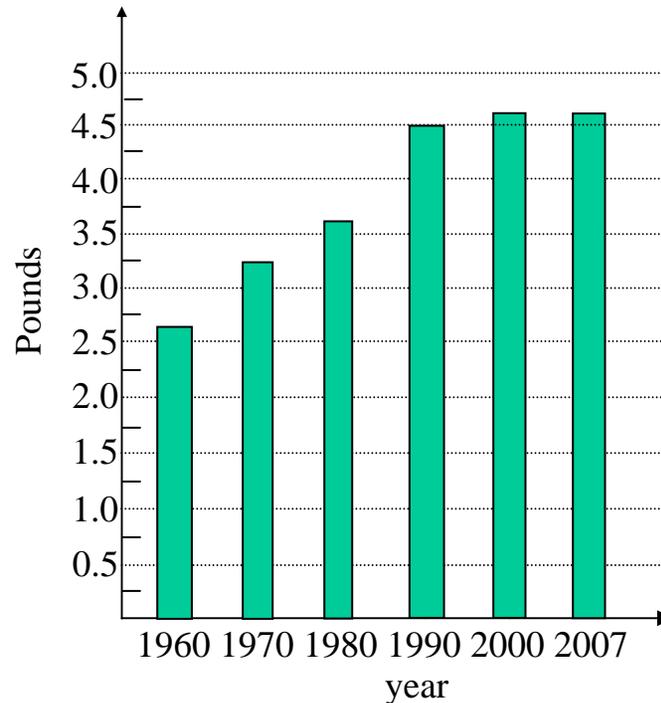
5.1 An Introduction to the Decimals

7. Read tables and graphs involving decimals

The table shows the number of pounds of trash generated daily per person in the U.S. for selected years from 1960 through 2007.

Year	pounds
1960	2.68
1970	3.25
1980	3.66
1990	4.50
2000	4.64
2007	4.62

Pound of trash generated daily (per person)

5.2 Adding and Subtracting Decimals

1. Add decimals

2. Subtract decimals

3. Add and subtract signed decimals

4. Estimate sums and differences of decimals

5. Solve application problems by adding and subtracting decimals

5.2 Adding and Subtracting Decimals

1. Add decimals (review)

e.g.1 Add: 36.824 + 8.5 + 79 + 12.65   Ans. 136.974

2. Subtract decimals (review)

e.g.2 Subtract: 356.4 – 123.6   Ans. 232.8

e.g.3 Subtract 42.318 from 45.6

Ans. 3.282

5.2 Adding and Subtracting Decimals

3. Add and subtract signed decimals

Sign rule is exactly the same as in integer case!

e.g.4   Add: $-3.4 + (-8.3)$ Ans. -11.7

e.g.5 Add: $15.21 + (-29.8)$ Ans. -14.59

e.g.6 Subtract: $-46.8 - 7.6$ Ans. -54.4

5.2 Adding and Subtracting Decimals

3. Add and subtract signed decimals

Sign rule is exactly the same as in integer case!

e.g.7 Subtract: $-2.98 - (-27.3)$ Ans. 24.32

e.g.8 Evaluate: $-33.5 - (-16.3 + 7.9)$ Ans. -25.1

5.2 Adding and Subtracting Decimals

4. Estimate sums and differences of decimals

e.g.9 Estimate

a. Estimate by rounding the addends to the nearest ten:

$\qquad$ 659.59 + 223.75 $\qquad$ *Ans*. 880

b. Estimate by front-end rounding:

$\qquad$ 734.01 – 48.22 $\qquad$ *Ans*. 650

Note: front-end rounding means that each number is
$\qquad$ rounded to its largest place value.

5.2 Adding and Subtracting Decimals

5. Solve application problems by adding and subtracting decimals

e.g.10 PRICING Find the retail price of an electric stapler if an office supply company buys them for $29.50 each and then marks them up $12.95 to sell in its stores.

Ans. $42.45

Note: retail price = cost + markup

5.2 Adding and Subtracting Decimals

5. Solve application problems by adding and subtracting decimals

e.g.11 TABLES A kitchen table has a 20-gauge stainless steel top that is 0.0375 inch thick and legs made of 16-gauge stainless steel that is 0.0625 in thick. How much thicker is the 16-gauge stainless steel?

Ans. 0.025 inch

5.2 Adding and Subtracting Decimals

5. Solve application problems by adding and subtracting decimals

e.g.12 CONDITIONING PROGRAM A 180-pound baseball player lost 4.2 pounds during the first week of spring training. He gained back 1.9 pounds the second week. Find his weight after two weeks of spring training.

Ans. 177.7 lb

5.3 Multiplying Decimals

1. Multiplying decimals (review)

2. Multiply decimals by power of 10 (review)

3. Multiply signed decimals

4. Evaluate exponential expression that have decimal base

5. Use the order of operations rule

6. Evaluate formulas

7. Estimate product of decimals

8. Solve application problems by multiplying decimals

5.3 Multiplying Decimals

1. Multiplying decimals (review)

e.g.2 Multiply: 0.004(2.3)   Ans. 0.0092

e.g.3 Multiply: 286(3.9)   Ans. 1,115.4

2. Multiply decimals by powers of 10 (review)

Comment: when multiplying a decimal by 10, 100, 1000, and so on, move decimal point to right

e.g.4 Multiply:

a.   3.725(100)   b.   46.3(100,000)

      Ans. 372.5   Ans. 4,630,000

5.3 Multiplying Decimals

2. Multiply decimals by powers of 10 (review)

Comment: when multiplying a decimal by 0.1, 0.01, 0.001, and so on, move decimal point to left

e.g.4 Multiply: Ans.   0.05426

a.   542.6(0.0001)   Ans.   0.0083

b.   0.83(0.01)

5.3 Multiplying Decimals

2. Multiply decimals by powers of 10 (review)

What is million? (10^6), billion? (10^9), and trillion? (10^{12})

Write each number in standard notation:

a. In 2007, the U.S had 930.9 million acres of farmland.

Ans.   930,900,000

b. Americans took 10.7 billion trips on public transportation in 2008.   Ans.   10,700,000,000

c. It would take about 1.818 trillion pennies to fill the Empire State Building.   Ans.   1,818,000,000,000

5.3 Multiplying Decimals

3. Multiply signed decimals

Sign rule is exactly the same as in integer case!

e.g.7 Multiply

a.   –3.6(2.8)   Ans.   –10.08

b.   (–10,000)(–63.11)   Ans.   631,100

5.3 Multiplying Decimals

4. Evaluate exponential expression that have decimal base (like integers)

e.g.8 Evaluate:

a. $(4.2)^2$

b. $(-0.07)^2$ Ans.   0.0049

5.3 Multiplying Decimals

5. Use the order of operations rule

(like integers)

e.g.9 Evaluate: $(-0.9)^2 + 4\,|-7.1 + 6.8\,|$ Ans.   2.01

6. Estimate Formulas (review)

e.g.10 Evaluate: $A = P + Prt$ for $P = 75.50$, $r = 0.12$, $t = 2$

Ans.   93.62

5.3 Multiplying Decimals

7. Estimate product of decimals

e.g.11 Estimate product.

a. Estimate using front-end rounding 2.371(76)   Ans. 160

b. Evaluate by rounding all factors to the nearest tenth:

   9.1682(7.344)   Ans. 67.16

c. Estimate by rounding: 0.5301(97.89) Ans. 53.01

 (hint: $97.89 \approx 100$)

5.3 Multiplying Decimals

8. Solve application problems by multiplying decimals

e.g.12 COINS Banks wrap dimes in rolls of 50 cents. If a dime is 1.35 millimeters thick, how tall is a stack of 50 dimes?   Ans.   67.5 mm

e.g.13 WEEKLY EARNINGS An iron worker's basic workweek is 40 hours. After his daily shift is over, he can work overtime at a rate of 1.5 times his regular rate of $18.08 per hour. How much money will he earn in a week if he works 2hours of overtime?   Ans. $777.44

5.4 Dividing Decimals

1. Divide a decimal by a whole number
2. Divide a decimal by a decimal
3. Round a decimal quotient
4. Estimate quotients of decimals
5. Divide decimals by power of 10
6. Divide signed decimals
7. Use the order of operations rule
8. Evaluate formulas
9. Solve application problems by dividing decimals

5.4 Dividing Decimals

1. Divide a decimal by a whole number (review)

e.g.2 Divide: 88.32 ÷ 24   Ans.   3.68

e.g.3 Divide: 37.4 ÷ 4   Ans.   9.35

2. Divide a decimal by a decimal (review)

e.g.4 Divide: 0.1596 ÷ 0.38   Ans.   0.42

Question: how to check the result?

5.4 Dividing Decimals

3. Round a decimal quotient (review)

e.g.5 Divide: $11.07 \div 0.7$, round the quotient to the nearest hundredth   Ans.   15.81

4. Estimate quotients of decimals

Rule of thumb: round both up or round both down

e.g.6 Estimate the quotient: $481.797 \div 62.98$ Ans.   8

5.4 Dividing Decimals

5. Divide decimals by power of 10

Comment:

– when dividing a decimal by 10, 100, 1000, and so on, move decimal point to left

– when dividing a decimal by 0.1, 0.01, 0.001, and so on, move decimal point to right

e.g.7 Find each quotient:

a. $721.3 \div 100$ b. $\frac{1.67}{1,000}$ c. $49.87513 \div 0.0001$

Ans. 0.00167   Ans. 498,751.3

5.4 Dividing Decimals

6. Divide signed decimals

Sign rule: just like dividing integers

e.g.8 Divide:

a. $-113.016 \div 13.6$ Ans. -8.31

b. $\dfrac{-31.93}{-0.1}$ Ans. 319.3

5.4 Dividing Decimals

7. Use the order of operations rule

Just like integers

e.g.9 Evaluate: $\dfrac{2(0.932) + 0.6764}{0.3 - 0.9}$

Ans.   –4.234

8. Evaluate formulas (review)

e.g.10 Evaluate the formula $h = \dfrac{3V}{B}$ for $V = 3.144$ and

$B = 2.4$

Ans.   3.93

5.4 Dividing Decimals

9. Solve application problems by dividing decimals

Like review

e.g.11 PEPPERONI A meat slicing machine cuts 12-inch-long pepperoni sausages into 0.125-inch-thick slices. How many slices are there in one sausage?

Ans.   96 slices

5.4 Dividing Decimals

9. Solve application problems by dividing decimals

e.g.12 BROADWAY SHOWS Use the following data to determine the average number of tickets that were sold per year to Broadway shows in New York City for years 2004 through 2008.

season	tickets sold (in millions)
2008	12.32
2007	12.29
2006	12.00
2005	11.53
2004	11.61

Ans.   11.95 millions

5.5 Fractions and Decimals

1. Write fractions as equivalent terminating decimals
2. Write fractions as equivalent repeating decimal
3. Round repeating decimals
4. Graph fractions and decimals on a number line
5. Compare fractions and decimals
6. Evaluate expressions containing fractions and decimals
7. Solve application problems involving fractions and decimals

5.5 Fractions and Decimals

1. Write fractions as equivalent terminating decimals

2. Write fractions as equivalent repeating decimal

This is review

To write fractions as decimal, just divide. The quotient is either terminating decimal or repeating decimal.

e.g.1 b. Write $\dfrac{7}{8}$ as a decimal.   Ans.   0.875

e.g.3 Write mixed number $1\dfrac{15}{16}$ in decimal form. Ans. 1.9375

e.g.5 Write $-\dfrac{13}{33}$ as a decimal.   Ans. $-0.\overline{39}$

5.5 Fractions and Decimals

3. Round repeating decimals (review)

e.g.7 Write $\dfrac{11}{12}$ as a decimal and round to the nearest thousandth.   Ans. 0.917

4. Graph fractions and decimals on a number line

Change fraction to decimal then plot all decimal points.

5.5 Fractions and Decimals

5. Compare fractions and decimals (review)

Change fraction to decimal

e.g.8 Place an <, > or = symbol in the box to make a true statement.

a. $\dfrac{3}{5}$ $\boxed{<}$ 0.66   b. 0.67 $\boxed{>}$ $\dfrac{2}{3}$ c. $\dfrac{9}{8}$ $\boxed{=}$ 1.125

e.g.9 Write the numbers in order from smallest to largest:

$3.584, \quad 3\dfrac{7}{12}, \quad \dfrac{18}{5}$

Ans. $3\dfrac{7}{12}, \quad 3.584, \quad \dfrac{18}{5}$

5.5 Fractions and Decimals

6. Evaluate expressions containing fractions and decimals (partly review)

e.g.10 Evaluate $\boxed{\dfrac{2}{3} + 0.11}$ by working in terms of fractions.

$$\boxed{\text{Ans. } \dfrac{233}{300}}$$

e.g.11 Evaluate $\dfrac{2}{3} + 0.11$ by working in terms of decimals.

Ans. ≈ 0.78

e.g.12 Evaluate: $\dfrac{3}{5}(1.75) + (0.6)^2$ Ans.   1.41

5.5 Fractions and Decimals

7. Solve application problems involving fractions and decimals

e.g.13 SHOPPING A shopper purchased $\frac{2}{3}$ pound of fruit, priced at $0.99 per pound, and $\frac{1}{2}$ pound of fresh-ground coffee, selling for $7.60 per pound. Find the total cost of these items.

Ans. $4.46

5.6 Square Roots

1. Find the square root of a perfect square
2. Find the square root of fractions and decimals
3. Evaluate expressions that contain square roots
4. Evaluate formulas involving square roots
5. Approximate square roots

5.6 Square Roots (very new)

1. Find the square root of a perfect square

A **square root** of a given number is a number whose square is the given number.

e.g.1 Find two square roots of 4.   Ans.   2 and –2

why?

In this example, we denote that

$$\sqrt{4} = 2$$

In word, the positive square root of 4 is 2.

5.6 Square Roots (very new)

1. Find the square root of a perfect square

In general, given a positive number *a*, the positive square root of *a* is denoted by

$$\sqrt{a}$$

and the negative square root of *a* is denoted by

$$-\sqrt{a}$$

e.g. Find square root of 5.   Ans.   No explicit solutions.

However we denote square roots of 5 by, $\sqrt{5}$ and $-\sqrt{5}$

e.g. Find square root of 8.   Ans.   Again no explicit solutions.

we denote square roots of 8 by, $\sqrt{8}$ and $-\sqrt{8}$

5.6 Square Roots (very new)

1. Find the square root of a perfect square

e.g. Find square root of 9.   Ans.   3 and –3

We see that only perfect squares have explicit square roots.
The following are first 16 perfect squares:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

e.g.2 Evaluate each square root:

a. $\sqrt{25}$ Ans. 5   b. $-\sqrt{121}$ Ans. –11

5.6 Square Roots (very new)

1. Find the square root of a perfect square

Expression $\sqrt{4}$ represents the positive square root of 4, here

- $\sqrt{4}$ is called radical expression
- 4 is called radicand
- $\sqrt{}$ is called radical symbol

5.6 Square Roots

2. Find the square root of fractions and decimals

e.g.3 Evaluate each square root:

a. $\sqrt{\dfrac{81}{4}}$ Ans. $\dfrac{9}{2}$ b. $\sqrt{0.25}$ Ans. 0.5

5.6 Square Roots

3. Evaluate expressions that contain square roots

e.g.4 Evaluate:

a. $\sqrt{81} + \sqrt{144}$ Ans. 21   b. $-\sqrt{169} - \sqrt{100}$ Ans. -23

e.g.5 Evaluate:

a. $5\sqrt{49}$ Ans. 35   b. $-6\sqrt{1} + \sqrt{64}$ Ans. 2

e.g.6 Evaluate: $15 + 2[4^2 - (4 + 1)\sqrt{25}\,]$

Ans. -3

5.6 Square Roots

4. Evaluate formulas involving square roots

e.g.7 Evaluate $c = \sqrt{a^2 + b^2}$ for $a = 3$ and $b = 4$.   Ans. –3

5.6 Square Roots

5. Approximate square roots (use calculator)

e.g.8 Use calculator to approximate each square root.
Round to the nearest hundredth.

a. $\sqrt{217}$ b. $\sqrt{89.1}$ c. $\sqrt{0.032}$

Ans. 14.73   Ans. 9.44   Ans. 0.18

5.7   Solving Equations that Involve Decimals

1. Simplify products
2. Use the distributive property
3. Simplify expressions by combining like terms
4. Use one property of equality to solve equations that involve decimals
5. Use more than one property of equality to solve equations that involve decimals
6. Use equations to solve application problems that involve decimals

5.7   Solving Equations that Involve Decimals

1. Simplify products (Like integers)

e.g.1 Multiply

a. $7.3 \cdot 4y$ Ans. $29.2y$

b. $6.52b(-3)$ Ans. $-19.56b$

c. $-2.8(-7.7r)$ Ans. $21.56r$

d. $3(5.09n)2$ Ans. $30.54n$

5.7   Solving Equations that Involve Decimals

2. Use the distributive property (Like integers)

e.g.2 Multiply

a. $7.3(8t + 9)$ Ans. $56.4t + 65.7$

b. $-5(8.5n - 6.9)$ Ans. $-42.5n + 34.5$

c. $(m - 2.4)0.09$ Ans. $0.09m - 0.216$

5.7   Solving Equations that Involve Decimals

3. Simplify expressions by combining like terms

(Like integers)

e.g.3 Simplify by combining like terms

a. $4.5a + 3.3a$ b. $-5.1b + (-9.7b) + 4.9b$

   Ans. $7.8a$ Ans. $-9.9b$

c. $0.33h^2 - 0.87h^2$ d. $5.02y + 6 + 2.97y - 1.31$

   Ans. $-0.54h^2$ Ans. $7.99y + 4.69$

e.g.4 Simplify: $3.8(y + 2) - 4.9 - (1.4y + 7.9)$

          Ans. $2.4y - 5.2$

5.7 Solving Equations that Involve Decimals

4. Use one property of equality to solve equations that involve decimals (Like integers)

e.g.5 Solve each equation and check the result:

a. $x + 3.7 = 10.9$ b. $x - 2.31 = -5.68$

     Ans. $x = 7.2$ Ans. $x = -3.37$

e.g.6 Solve: $\dfrac{x}{4} = -9.9$ Ans. $x = -39.6$

e.g.7 Solve: $-24.08 = -5.6x$ Ans. $x = 4.3$

5.7 Solving Equations that Involve Decimals

5. Use more than one property of equality to solve equations that involve decimals (Like integers)

e.g.8 Solve: $7.1m - 1.03 = -6.71$ Ans. $m = -0.8$

e.g.9 Solve the equation and check the result:

$$0.1m - 4 = 0.9m + 1.6$$ Ans. $m = -7$

e.g.10 Solve: $5(m + 1.9) + 6.3 = -11.8$ Ans. $m = -5.52$

5.7   Solving Equations that Involve Decimals

6. Use equations to solve application problems that involve decimals(Like integers)

e.g.11 GRADUATION A printer charges a setup fee of $16 and then 85 cents for each graduation announcement printed (tax included). If a student wants to spend $50 on her announcements, how many can she have printed?

Ans. 40 graduation announcements

Note: 5 step process.