The art in mathematics

Graphing was used in the ancient world by the Egyptians and the Romans in surveying and by the Greeks in mapmaking. However, it was not until the 14th century that graphing as we now know it had its origins in the works of the French mathematician Nicole Oresme. It was the one mathematical bright spot in that century of the Black Death and the Hundred Years War.

The idea of graphing with coordinate axes (pronounced ax-eeze) dates both to Descartes in the 17th century and before him to Apollonius in the 2nd century B.C. Graphing did not appear in the high school textbooks of the early 1900s. By the 1930s, however, even junior high teachers considered graphing to be an important part of algebra, one that could not easily be omitted. The rapid growth in its general importance was probably due to the use of graphic representation in statistics to show how one variable is dependent on another. Graphing is also an important part of navigation and of modern warfare.

A graph is a drawing that is intended to make a collection of data or a mathematical expression easier to understand. However, for many people the graph does nothing of the sort—it only serves to confuse. This chapter will try to change this.

8.1 DRAWING GRAPHS

To draw a graph yourself, start by drawing two lines, called axes, one horizontal and one vertical. You must decide what information should go along the horizontal axis and what should go along the vertical axis (Figure 8.1).
EXAMPLE

Draw a graph for the following data, which show the temperature for the week beginning January 1.

<table>
<thead>
<tr>
<th>Date</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1</td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>27</td>
</tr>
<tr>
<td>6</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
</tr>
</tbody>
</table>

Solution We usually place along the horizontal axis the numbers that change at a regular rate (the independent variable), in this case the days. Along the vertical axis we place the values that depend on the given value of the independent variable. These we call the dependent variable. In this case the temperature is the dependent variable because its value depends on which day the temperature was measured.

In this case the dependent numbers range from 9 to 40. Rather than filling in every number from 9 to 40, we could use intervals such as 5, 10, 15, . . ., 40 to represent this range. We write the dates along the horizontal axis, evenly spaced. Label both axes (Figure 8.2).

To mark the temperature for January 1 we go to 1 on the horizontal axis. The temperature on January 1 was 35°F, so we draw a point that is directly above 1 on the date axis and opposite 35 on the temperature axis (Figure 8.3). This point represents the temperature on January 1. For January 2, we mark a point above 2 and opposite where 24 would be on the vertical axis. We continue marking points for the other dates until the graph looks like the one to the left in Figure 8.3. As a final step, we can connect the points to show how the temperature changed for the week of January 1 (right in the figure).

What conclusions can you draw about the temperature during the first week of January?
EXAMPLE

Plot the following student test scores.

<table>
<thead>
<tr>
<th>Test</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>72</td>
</tr>
<tr>
<td>2</td>
<td>81</td>
</tr>
<tr>
<td>3</td>
<td>96</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
</tr>
<tr>
<td>5</td>
<td>92</td>
</tr>
</tbody>
</table>

Solution Here the low score is 43 and the high score is 96; a good choice of intervals for the vertical scale would be 10, 20, 30, ..., 100. The test numbers are the independent variable, so we list them across the horizontal axis. The average test score is the dependent variable—the score depends on which test we are looking at. We list the test scores along the vertical axis. See Figure 8.4.

The average score for the first test is marked above the 1 (identifying the test) and across from where 72 would be on the vertical axis (identifying the score). Other scores are marked in a similar way.
The choice of scales is up to you. The shape of the graph changes with
different scales as you can see in the two graphs shown in Figure 8.4.
What conclusions can you draw from these graphs?

EXERCISE 8.1.1

1. A newspaper article reported on how the value of a certain stock fluctuated
during the first 9 months of the year. The values are listed in Table 8.1. Use
the values from the table to draw a graph. Label the horizontal axis “Month”
and the vertical axis “Price per share ($).” What conclusions can you draw
from the graph about how the stock fluctuated?

2. Experiments in science often give values that are plotted on a graph to clarify
relationships between sets of numbers. For instance, the pressure of a confined
gas increases when the temperature increases. Table 8.2 shows this relationship.
Use a suitable scale and graph the pressure as a dependent variable. The
temperature (the independent variable) should be plotted along the horizon­
tal axis and the pressure along the vertical axis. Conclusion: If you heat a gas­
filled bottle it may eventually explode!

<table>
<thead>
<tr>
<th>Month</th>
<th>Price per share ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>26</td>
</tr>
<tr>
<td>February</td>
<td>30</td>
</tr>
<tr>
<td>March</td>
<td>28</td>
</tr>
<tr>
<td>April</td>
<td>26</td>
</tr>
<tr>
<td>May</td>
<td>30</td>
</tr>
<tr>
<td>June</td>
<td>24</td>
</tr>
<tr>
<td>July</td>
<td>28</td>
</tr>
<tr>
<td>August</td>
<td>26</td>
</tr>
<tr>
<td>September</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Pressure (mm Hg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15</td>
<td>1</td>
</tr>
<tr>
<td>-10</td>
<td>2</td>
</tr>
<tr>
<td>-5</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>25</td>
<td>24</td>
</tr>
</tbody>
</table>

3. Given the data below, showing the yearly income of the ABC corporation,
draw the corresponding graph.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Income ($Million)</td>
<td>0.56</td>
<td>0.50</td>
<td>0.20</td>
<td>0.29</td>
<td>0.89</td>
<td>3.23</td>
</tr>
</tbody>
</table>
CHAPTER 8  GRAPHING

8.2  GRAPHING WITH ORDERED PAIRS

Martin and Nancy go shopping and spend exactly the same amount of money at each store. At the first store each spends $1; at the second, $4; and at the third, $10.

We can draw a graph showing this information. First we make a table of the given facts:

<table>
<thead>
<tr>
<th>Martin</th>
<th>Nancy</th>
<th>(M, N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$1</td>
<td>(1, 1) (over 1, then up 1)</td>
</tr>
<tr>
<td>$4</td>
<td>$4</td>
<td>(4, 4) (over 4, then up 4)</td>
</tr>
<tr>
<td>$10</td>
<td>$10</td>
<td>(10, 10) (over 10, up 10)</td>
</tr>
</tbody>
</table>

Then we draw and label the axes and graph the points as in Figure 8.5.

![Figure 8.5](image)

EXAMPLE

This time, at each store, Nancy spends a dollar more than Martin. For example, when Martin spends $1, Nancy spends $2; when Martin spends $2, Nancy spends $3. Revise your graph for this new information. See Figure 8.6.

Solution  Make a table of these facts before you draw the graph.

<table>
<thead>
<tr>
<th>Martin</th>
<th>Nancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
Do you see that these points lie in a straight line? Connect the points. Which values would you read for A? for B?

Point A: Martin $6, Nancy $7
Point B: Martin $8, Nancy $9

EXAMPLE

This time Nancy spends twice what Martin does. Make the new table and draw the graph. See Figure 8.7.
Solution  Let $M$ stand for what Martin spends and $N$ for what Nancy spends.

\[
\begin{array}{c|c}
M (\$) & N (\$) \\
\hline
1 & 2 \\
2 & 4 \\
3 & 6 \\
4 & 8 \\
\end{array}
\]

$N = 2M$ expresses in symbols that Nancy's spending is twice Martin's.

EXAMPLE

Nancy spends $3$ more than Martin each time. Prepare the table and draw the graph.

Solution  Written in short form: $N = M + 3$.

\[
\begin{array}{c|c|c}
M & N & (M, N) \\
\hline
1 & 4 & (1, 4) \\
2 & 5 & (2, 5) \\
3 & 6 & (3, 6) \\
4 & 7 & (4, 7) \\
\end{array}
\]

Notice that this time beside each point on the graph instead of writing (1 for Martin, 4 for Nancy) or even $M = 1$, $N = 4$, etc., we write (1, 4), (2, 5), (3, 6), and (4, 7). These are called ordered pairs or coordinates. The first number (the independent variable) is always the number on the horizontal axis, and the second number (the dependent variable) is always the number on the vertical axis. See Figure 8.8.
EXERCISE 8.2.1

Write an equation, construct a table, and draw the graph.

1. Nancy spends 3 times what Martin does.
2. Nancy spends $5 more than Martin does.
3. Nancy spends $1 less than Martin does.
4. Nancy spends $3 less than Martin does.
5. Nancy spends $3 more than twice what Martin spends.

Graphing Ordered Pairs

In order to generalize coordinate graphing, draw two axes, one vertical and one horizontal, so that they intersect at the point where both are zero. Positive numbers will be above and to the right of this intersection. Negative numbers will be below and to the left of the intersection. The intersection itself is called the origin. The number for the horizontal number line is listed first.

EXAMPLE

Plot the ordered pairs A (1, 2), B (3, 5), C (-2, 3), D (-3, -4), E (2, -1), and F (0, 5).

Solution

See Figure 8.9.

A (1, 2): Start at the origin, move 1 step to the right and continue 2 steps up.
B (3, 5): Start at the origin, move 3 steps to the right and 5 steps up.
C (-2, 3): Start at the origin, move 2 steps to the left and 3 steps up.
D (-3, -4): Start at the origin, move 3 steps to the left and 4 steps down.

Figure 8.9
CHAPTER 8 GRAPHING

E (2, -1): Start at the origin, move 2 steps to the right and 1 step down.
F (0, 5): Start at the origin, move 0 steps to the right and 5 steps up.

EXERCISE 8.2.2

1. Draw two intersecting axes, as for Figure 8.9. Locate and label the following points:

   A (1, 1)  D (-1, 4)  G (2, 0)  J (0, 0)
   B (2, 6)  E (1, -4)  H (3, -2)  K (-1, -3)
   C (6, 2)  F (-3, -5) I (0, 4)  L (-3, 0)

2. Find the ordered pair that goes with each of the letters in Figure 8.10.

8.3 GRAPHINGRELATED PAIRS

So far the graphs in this chapter have related years and income and two people’s spending habits. Thus far we have used various labels for our number lines. Often, however, the letters x and y are used to stand for whatever pair of ideas we are relating. The horizontal axis shows the x-values, and the vertical axis shows the y-values. Now, instead of \( N = M \), we have \( y = x \); instead of \( N = M + 3 \), we have \( y = x + 3 \). Ordered pairs always list the x-value first. The x-value is the independent variable, and the y-value is the dependent variable (y depends on the value assigned to x).

Now, suppose we have the statement “y is 3 more than x” to graph. We rewrite the statement as an equation, \( y = 3 + x \) or \( y = x + 3 \). Then we find ordered pairs \((x, y)\) by choosing values for \( x \) and then solving the equation to find the values for \( y \).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>(1, 4)</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>(2, 5)</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>(3, 6)</td>
</tr>
</tbody>
</table>
Let's try some negative values for $x$: 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2</td>
<td>(-1, 2)</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
<td>(-3, 0)</td>
</tr>
<tr>
<td>-6</td>
<td>-3</td>
<td>(-6, -3)</td>
</tr>
</tbody>
</table>

If $x = -1$, then $y = -1 + 3 = 2$.
If $x = -3$, then $y = -3 + 3 = 0$.
If $x = -6$, then $y = -6 + 3 = -3$.

All these points lie on one line. The equation $y = x + 3$ implies that for every $x$-value, there is only one $y$-value. In order for the graph to form a curve, there must be more than one $y$-value for some $x$-values or more $x$-values for some $y$-values. A graph will also curve if one variable is an exponent.

When we graph a line, it is best to locate at least three points. If they lie in a straight line we know that we have not made a mistake in our substitution. We could have used any three pairs. If we use $(2, 5)$, $(3, 6)$, and $(-6, -3)$, we will have exactly the same line as if we had used $(1, 4)$, $(3, 6)$, and $(-1, 2)$. (See Figure 8.11.)

![Figure 8.11](image)

**EXAMPLE**

Use the process just described to graph the statement "$y$ is 3 times as large as $x$.”

**Solution** We begin by translating the statement into an equation.

$$y = 3x$$

Next make the table of values for which $y = 3x$ is true.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Now we can write the ordered pairs (1, 3), (2, 6), (3, 9), and (0, 0) and locate them as points on a graph. Last, we draw a line connecting the points in order. (See Figure 8.12.) This line is the graph of $y = 3x$. 
Look at the equation \( y = \frac{1}{3}x \).

Again we can substitute any values we want for \( x \). Let's try 1, 2, 3:

\[
\begin{array}{c|c}
  x & y \\
  \hline
  1 & 1/3 \\
  2 & 2/3 \\
  3 & 3/3 \text{ or } 1 \\
\end{array}
\]

(1, 1/3) and (2, 2/3) are harder to graph because of the fractions that represent the \( y \) value. Let's try to find values for \( x \) that will lead to whole-number values for \( y \).

\[
\begin{array}{c|c}
  x & y \\
  \hline
  0 & 0 \\
  3 & 1 \\
  6 & 2 \\
\end{array}
\]

For the equation \( y = \frac{1}{3}x \), any multiple of 3 for the \( x \) value will lead to a whole-number value for \( y \). Here are some more:

\[
\begin{array}{c|c}
  x & y \\
  \hline
  9 & 3 \\
  -3 & -1 \\
  -6 & -2 \\
\end{array}
\]

We can now plot three of the ordered pairs and draw the line connecting them. Figure 8.13 shows the result using \((-3, -1), (3, 1), \) and \((6, 2)\).
EXAMPLE
Graph \( y = \frac{2}{3}x \).

Solution This time, to avoid fractions we use multiples of 5 for the \( x \) value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

Plotting the points from the table and connecting them leads to the graph of \( y = \frac{2}{3}x \) shown in Figure 8.14.

Figure 8.14

EXAMPLE
Graph \( y = -x \).

Solution First make the table with three values of your choice for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

[Remember, \(-(-1) = 1\)]

Then plot the points and connect them. (See Figure 8.15.)
EXAMPLE

Draw the graph that corresponds to the statement “y is 3 more than 2 times x.”

Solution The equation is $y = 2x + 3$.

Substitute values for $x$ to find corresponding values for $y$. Make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

If $x = 0$, then $y = 2(0) + 3 = 0 + 3 = 3$.

If $x = 1$, then $y = 2(1) + 3 = 5$.

If $x = 2$, then $y = 2(2) + 3 = 7$.

The ordered pairs $(x, y)$ are $(0, 3), (1, 5), (2, 7)$. These points are used to graph the line $y = 2x + 3$ in Figure 8.16.

EXAMPLE

Graph $y = 3x - 5$.

Solution Find three points, plot them, and connect the points.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

If $x = 0$, then $y = 3(0) - 5 = -5$, giving the point $(0, -5)$.

If $x = 1$, then $y = 3(1) - 5 = -2$, giving the point $(1, -2)$.

If $x = 2$, then $y = 3(2) - 5 = 1$, giving the point $(2, 1)$.

These points are used in Figure 8.17 to graph $y = 3x - 5$. 
EXAMPLE

Graph \( y = -\frac{1}{2}x + 1 \).

**Solution**  Make a table with three ordered pairs. Plot the points and connect them.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>(2, 0)</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>(4, -1)</td>
</tr>
</tbody>
</table>

The graph is shown in Figure 8.18.
Suppose the equation we want to graph is $y = 3$.

If $x = 0$, what is $y$? $y = 3$
If $x = 2$, what is $y$? $y = 3$
If $x = -2$, what is $y$? $y = 3$

List these values in a table and then draw the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>(2, 3)</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
<td>(-2, 3)</td>
</tr>
</tbody>
</table>

Figure 8.19 shows that the graph of $y = 3$ is a line through $(0, 3)$ parallel to the $x$ axis.
EXAMPLE

Graph \( y = -3 \).

**Solution**  Follow the same steps as before.

\[
\begin{array}{c|c|c}
\text{x} & \text{y} & (\text{x, y}) \\
\hline
-1 & -3 & (-1, -3) \\
0 & -3 & (0, -3) \\
5 & -3 & (5, -3) \\
\end{array}
\]

The line intersects the \( y \) axis at \((0, -3)\) and is parallel to the \( x \) axis, as shown in Figure 8.20.

![Figure 8.20](image)

EXAMPLE

Graph \( x = 2 \).

**Solution**  The equation states that \( x \) is always 2.

Make the table with all \( x \) entries equal to 2; \( y \) can be anything.

\[
\begin{array}{c|c|c}
\text{x} & \text{y} & (\text{x, y}) \\
\hline
2 & 0 & (2, 0) \\
2 & 8 & (2, 8) \\
2 & -3 & (2, -3) \\
\end{array}
\]

The line intersects the \( x \) axis at \((2, 0)\) and is parallel to the \( y \) axis. (See Figure 8.21.)

All of the equations discussed above are called *linear* equations because their graphs are *straight* lines.
EXERCISE 8.3.1

Find three points that satisfy the given equation. Plot these points, and draw the line.

1. \( y = 2x \)
2. \( y = x + 4 \)
3. \( y = -\frac{1}{3}x \)
4. \( y = x - 4 \)
5. \( y = 2x + 1 \)
6. \( y = 3x - 4 \)
7. \( y = \frac{2}{3}x + 1 \)
8. \( y = -x + 5 \)
9. \( y = -3x \)
10. \( y = -3x + 1 \)
11. \( y = -x - 2 \)
12. \( y = \frac{1}{2}x \)
13. \( y = -2x + 5 \)
14. \( y = -2x + 3 \)
15. \( y = -2x - 1 \)
16. \( y = -\frac{1}{2}x - 2 \)
17. \( y = 2 \)
18. \( x = -3 \)
19. \( y = -3 \)
20. \( x = 7 \)
8.4 SLOPE AND $y$-INTERCEPT

Slope

Let’s look at some more graphs of linear equations.

1. $y = x$

   
<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

   

2. $y = \frac{1}{2}x$

   
<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>
3. \( y = 4x \)

\begin{center}
\begin{tabular}{|c|c|}
\hline
\( x \) & \( y \) \\
\hline
0 & 0 \\
1 & 4 \\
2 & 8 \\
\hline
\end{tabular}
\end{center}

4. \( y = -4x \)

\begin{center}
\begin{tabular}{|c|c|}
\hline
\( x \) & \( y \) \\
\hline
0 & 0 \\
1 & -4 \\
2 & -8 \\
\hline
\end{tabular}
\end{center}

What do you see happening? Each of these lines goes through the point (0, 0), the origin.
In the first three graphs, as we look from left to right, the line goes up; in the fourth graph, the line goes down. The rise or fall of a graph line as you travel along it from left to right is called its slope. Each of these lines has a different slope.

When the coefficient of $x$ is positive ($1, \frac{1}{2}, 4$), the slope is positive, and the line goes up from left to right. The larger the coefficient, the steeper the slope will be. We see that the graph of $y = 2x$ rises more quickly than the graph of $y = \frac{1}{2}x$.

Similarly, when the coefficient of $x$ is negative, the slope is negative, and the line goes down from left to right. Its steepness depends on the size of the absolute value of the coefficient. Remember that the absolute value is the positive magnitude of the number.

$|3| = 3, |-3| = 3$.

How do we define slope? We can use the carpenter's definition, 

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

Rise is the vertical change; run is the horizontal change.

The slope of the flight of stairs in Figure 8.22 is the rise (6 ft) divided by the run (10 ft). The steps rise from left to right, so the slope is positive.

$$\text{Slope} = \frac{6}{10} = \frac{3}{5}$$

![Figure 8.22](image)

The slope of a roof is positive or negative, depending on which part of the roof we are looking at. In part a of the roof in Figure 8.23, the vertical change is $+8$, and the horizontal change is 12.

$$\text{Slope} = \frac{8}{12} = \frac{2}{3}$$

In part b, the vertical change is $-8$, leading to a slope of $\frac{-8}{12} = -\frac{2}{3}$.

![Figure 8.23](image)
EXAMPLE

Find the slope of the line connecting the two points \((0, 0)\) and \((2, 8)\).

**Solution**  First we plot the two points (Figure 8.24).
The rise is 8 and the run is 2. Therefore,

\[
\text{Slope} = \frac{8}{2} = 4
\]

![Figure 8.24](image)

EXAMPLE

Find the slope of the line connecting \((0, 0)\) and \((3, -4)\).

**Solution**  The two points are plotted and a line is drawn through them in Figure 8.25.

\[
\text{Rise} = -4, \quad \text{run} = 3, \quad \text{slope} = -\frac{4}{3}.
\]

EXAMPLE

Find the slope of the line containing \((1, 4)\) and \((8, 7)\).

**Solution**  See Figure 8.26.
Rise (the change in the value of \(y\)) = 3
Run (the change in the value of \(x\)) = 7

\[
\text{Slope} = \frac{3}{7}
\]
Suppose we measured from $(8, 7)$ to $(1, 4)$ in Figure 8.26. The change in $y$ would be $-3$, while the change in $x$ would be $-7$.

$$\text{Slope} = \frac{-3}{-7} \text{ or } \frac{3}{7}$$

It makes no difference in which order we read the points as long as we read the changes in $y$ in the same order as the changes in $x$; the slope will still measure the change in the line as it goes from left to right.
In definitions and formulas, we sometimes refer to a point as \( P_1(x_1, y_1) \), which means that the point \( P_1 \) (pronounced "P sub one") has the coordinates \( x_1 \) and \( y_1 \).

**DEFINITION**

Given two points, \( P_1(x_1, y_1) \) and \( P_2(x_2, y_2) \), the slope of the line joining the two points is (see Figure 8.27)

\[
\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}
\]

The letter \( m \) is used to represent the slope of a line. A capital Greek letter delta, \( \Delta \), is used to represent change. Thus \( \Delta y = \text{change in } y \), and \( \Delta x = \text{change in } x \). So the definition of slope can also be written

\[
m = \frac{\Delta y}{\Delta x}
\]

**EXAMPLE**

Given the equation \( y = -3x + 1 \). Find the slope.

**Solution**  First we find any two points on the line. Let’s let \( x = 0 \) and 4.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-11</td>
</tr>
</tbody>
</table>
Then
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-11 - 1}{4 - 0} = \frac{-12}{4} = -3 \]

We could also have solved this by reversing the points:
\[ m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{1 - (-11)}{0 - 4} = \frac{12}{-4} = -3 \]

The slope is the coefficient of \( x \) when the equation is in the form
\[ y = mx + b \]

Here, \( m \) represents the slope of the line and \( b \) is a constant. In the equation, \( y = -3x + 1 \), \(-3\) is \( m \) and \( 1 \) is \( b \).

For the equation \( y = 5x + 2 \), the slope is \( m = 5 \), \( b = 2 \).

For the equation \( y = -x - 4 \), the slope is \( m = -1 \), \( b = -4 \).

The equation \( y = 3 \) can be thought of as \( y = 0x + 3 \). Here we can see that the slope is \( m = 0 \). A slope of 0 indicates that the line is horizontal.

To find the slope of the equation \( x = 1 \), take two points, let’s say \((1, 5)\) and \((1, 3)\). By definition,
\[ m = \frac{\Delta y}{\Delta x} = \frac{5 - 3}{1 - 1} = \frac{2}{0} \]

Since we cannot divide by zero, this slope is undefined. Such a slope indicates a vertical line.

**Intercept**

Let’s look at the three equations
1. \( y = 2x + 3 \)
2. \( y = 2x \)
3. \( y = 2x - 3 \)

and plot them on the same axes, as in Figure 8.28.

All three lines have the same slope: \( m = 2 \). When two or more lines have the same slope, we say they are **parallel**. They will not intersect no matter how far we extend them in either direction. Each of these lines crosses the \( y \)-axis at a different point. We call these points the **\( y \)-intercepts**, the point where \( x = 0 \).

In each of the three equations above, we will set \( x = 0 \) to find the \( y \)-intercept.

1. \( y = 2x + 3 \)
   \( y = 2(0) + 3 \)
   \( y = 3 \) \( y \)-intercept is \((0, 3)\).
2. \( y = 2x \)
   \( y = 2(0) \)
   \( y = 0 \) \( y \)-intercept is \((0, 0)\).
3. \( y = 2x - 3 \)
   \( y = 2(0) - 3 \)
   \( y = -3 \) \( y \)-intercept is \((0, -3)\).
EXAMPLE

Find the y-intercept: (a) \(y = 5x - 7\) (b) \(y = x + 0.03\)

Solution
(a) The y-intercept is \((0, -7)\).
(b) The y-intercept is \((0, 0.03)\).

To find the y-intercept of any line in the form \(y = mx + b\), we set \(x = 0\). Then

\[
\begin{align*}
y &= mx + b \\
y &= 0x + b \\
y &= b
\end{align*}
\]

Thus, in any line \(y = mx + b\), \(m\) is the slope and \((0, b)\) is the y-intercept.

EXERCISE 8.4.1

1. Find the y-intercept.
   (a) \(y = 2x + 3\)
   (b) \(y = \frac{1}{2}x - 4\)
   (c) \(y = -x + 6\)
   (d) \(y = -3x - 2\)
   (e) \(y = 2\)
   (f) \(y = -3\)

2. Find the slope.
   (a) \(y = 2x\)
   (b) \(y = -x + 3\)
8.5 GRAPHING USING SLOPE AND Y-INTERCEPT

We can use what we have learned about slope and intercept to graph linear equations in the form \( y = mx + b \).

**EXAMPLE**

Graph \( y = \frac{2}{3}x + 4 \).

**Solution** From the equation, we can see that \( m = \frac{2}{3} \) and \( b = 4 \). Thus the slope is \( \frac{2}{3} \), and the \( y \)-intercept is the point \((0, 4)\).

First, we locate the point \((0, 4)\) on the graph (Figure 8.29). From this point we use the slope to find a second point. With a slope of \( \frac{2}{3} \), the change in \( x \) is 3, and the change in \( y \) is 2. From \((0, 4)\) we move to the right 3 units and up 2 units to the point \((3, 6)\) or from \((0, 4)\) we can move up 2 units and to the right 3 units.

![Figure 8.29](image)

Now draw the line connecting the two points.

**EXAMPLE**

Graph \( y = -3x + 5 \).

**Solution** \( y = -3x + 5 \) is in the form \( y = mx + b \). Here \( m = -3 = -\frac{3}{1} \). So the slope has a rise of -3 and a run of 1.

\( b = 5 \); therefore the \( y \)-intercept is \((0, 5)\). Locate \((0, 5)\) on the graph. From this point the change in \( x \) is 1, so go right 1 unit. The change in \( y \) is -3, so go
down 3 units to point (1, 2). Now connect the two points. The completed graph is shown in Figure 8.30.

Note that \(-3 = \frac{3}{1}\), so the slope rise is 3 and the run is \(-1\). Therefore, if you begin at (0, 5) you can go up 3 units and then left 1 unit. If you like alphabetical order (first \(x\), then \(y\)), you can first go 1 unit to the left and then 3 units up. In either case you get a point on the line.

**EXAMPLE**

Graph \(y = -x - 2\).

**Solution**  This equation is also in the form \(y = mx + b\). Think of it as \(y = -1(x) + (-2)\). Here

\[
m = -1 = \frac{-1}{1} \\
and \\n\[b = -2\]

Thus the slope has a rise of \(-1\) and a run of \(1\), and the \(y\) intercept is \((0, -2)\). The graph is shown in Figure 8.31.

Suppose we are given the equation \(2y - x = 6\) to graph. To rewrite this in the form \(y = mx + b\), we must solve for \(y\).

\[
\begin{align*}
2y - x &= 6 \\
2y &= x + 6 \\
\frac{2y}{2} &= \frac{x + 6}{2} \\
y &= \frac{1}{2}x + 3
\end{align*}
\]

Add \(x\).

Divide by 2.

Simplify.

The equation is now in the form \(y = mx + b\). \(m = \frac{1}{2}, b = 3\).
EXAMPLE
Rewrite $\frac{1}{2}y + 3x - 4 = 0$ in the form $y = mx + b$.

Solution

\[
\begin{align*}
\frac{1}{2}y + 3x - 4 &= 0 \\
\frac{1}{2}y + 3x &= 4 \\
\frac{1}{2}y &= 4 - 3x \\
y &= 8 - 6x \\
m &= -6
\end{align*}
\]

EXERCISE 8.5.1
Find the slope and the y-intercept. Then graph the line.

1. $y = x - 2$
2. $y = 5x$
3. $y = \frac{1}{2}x - 5$
4. $y = -3x - 1$
5. $y = 3x + 2$
6. $y + \frac{2}{3}x = 1$
7. $2y - 3x + 4 = 0$
8. $\frac{1}{4}x - \frac{3}{5}y = 4$
PART II  ALGEBRA

9.  $3x + 2y = 6$
10.  $2x - 5y = 10$

8.6 THE GRAPHING CALCULATOR

Certain calculators can plot graphs. One example is Texas Instruments TI-83 Plus. It is amazingly versatile but the instructions are very hard to follow. As an example, we will try to graph some functions.

EXAMPLE

On the same graph, plot $y = x$, $y = x + 2$, and $y = x + 4$.

Solution  Press the following keys: ON, MODE, Function, Y =. The calculator window will show Y_1 = and you press the key marked X, T, θ, n. Then go to the second line and press the key again and add + 2. Move to the third line and press the key and add + 4. Finally press GRAPH. You should now see three parallel lines one above the other.

There will be more examples in Chapter 14.

SUMMARY

Definition

Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

Linear equation: $y = mx + b$, where $m$ is the slope and $b$ is the y-intercept.

VOCABULARY

Coordinates: The two numbers in an ordered pair $(x, y)$. The first number is the x-coordinate; the second number the y-coordinate.

Delta ($\Delta$): A symbol used to signify a difference.

Dependent variable: The variable whose value depends on the value of another variable.

Graph: A picture signifying relationships between 2 sets of numbers.

Horizontal axis: The axis used for the independent variable.

Independent variable: A variable whose values can be chosen without regard to other variables.

Linear equation: An equation with two variables both to the first power. The slope form is $y = mx + b$.

Ordered pair: Two numbers $(x, y)$ that are written in order. Commonly used to identify a point on a graph.

Origin: The intersection of the x-axis and the y-axis.

Parallel lines: Lines with the same slope.

Related pairs: Two numbers that are connected by a rule.

Rise: Change in y between two points.

Run: Change in x between two points.

Slope: The ratio of the change in y to the change in x.

Vertical axis: The axis used for the dependent variable.

x-value: The value of the variable $x$; the first number in an ordered pair.

y-intercept: The point where a graph intersects the y axis.

y-value: The value of the variable $y$; the second number in an ordered pair.
CHECK LIST

Check the box for each topic you feel you have mastered. If you are unsure, go back and review.

☐ Ordered pairs and points
☐ Graphing ordered pairs
☐ Writing equations from rules
☐ Graphing related pairs
☐ y-intercept
☐ Slope
☐ Graphing vertical and horizontal lines
☐ Graphing lines using slope and $y$-intercept

REVIEW EXERCISES

1. Plot the following monthly average temperatures.

<table>
<thead>
<tr>
<th>Month</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>9</td>
</tr>
<tr>
<td>February</td>
<td>14</td>
</tr>
<tr>
<td>March</td>
<td>25</td>
</tr>
<tr>
<td>April</td>
<td>54</td>
</tr>
<tr>
<td>May</td>
<td>63</td>
</tr>
<tr>
<td>June</td>
<td>68</td>
</tr>
<tr>
<td>July</td>
<td>76</td>
</tr>
<tr>
<td>August</td>
<td>50</td>
</tr>
<tr>
<td>September</td>
<td>42</td>
</tr>
<tr>
<td>October</td>
<td>48</td>
</tr>
<tr>
<td>November</td>
<td>28</td>
</tr>
</tbody>
</table>

2. Write an equation, make a table of number facts, and plot the points.
   (a) Nancy spends 6 dollars more than Martin.
   (b) Nancy spends four times what Martin spends.
   (c) Nancy spends $3 less than Martin spends.
   (d) Nancy spends $1 more than two times what Martin spends.

3. Locate and label the ordered pairs.
   (a) $(2, 2)$  (b) $(2, -3)$  (c) $(-5, 1)$
   (d) $(-1, 0)$  (e) $(0, 0)$  (f) $(2, 0)$
   (g) $(0, -4)$  (h) $(-3, 0)$  (i) $(-1, -1)$

4. Express each point in Figure 8.32 as an ordered pair. For example, $A = (0, 0)$.

5. Find three points that satisfy the given equation. Plot the points, and draw the line.
   (a) $y = x + 2$
   (b) $y = x - 3$
   (c) $y = 3x$
   (d) $y = 3$
   (e) $y = 2x + 1$
   (f) $x = 4$
   (g) $y = -x + 2$
   (h) $y = 3x - 2$

6. Plot the two points and find the slope.
   (a) $(1, 3)(3, 6)$
   (b) $(2, -5)(8, 1)$
   (c) $(0, 6)(3, -1)$
   (d) $(4, 0)(0, 2)$
   (e) $(4, 2)(8, 2)$
   (f) $(2, 7)(7, 2)$

7. Find the slope and $y$-intercept. Graph the equation.
   (a) $y = x + 2$
   (b) $y = x - 5$
   (c) $y = 2x$
   (d) $y = -2x + 4$
   (e) $y = \frac{1}{2}x$
   (f) $y = -3$
   (g) $x + y = 6$
   (h) $2x - y = 2$
Referring to Figure 8.33, solve the problems to satisfy yourself that you have mastered Chapter 8.

1. Read point A.
2. Graph the point (2, -5).
3. Graph the line \( y = 2x + 4 \).
4. Graph the line \( 2x - 3y = 12 \).
5. Graph the line \( x = -6 \).
6. Graph the line \( y = 4 \).
7. What is the slope of the line \( y = 6x \)?
8. What is the \( y \)-intercept of the line \( y = 3x + 4 \)?
9. Find the slope of the line that connects (2, 3) and (4, 9).
10. Plot the following relationship using temperature as the independent variable:

\[
\begin{array}{c|c|c|c|c|c|c}
\text{Temperature} & -10^\circ & -5^\circ & 0^\circ & 5^\circ & 10^\circ & 15^\circ \\
\text{Volume} & 1 \text{ L} & 2 \text{ L} & 3 \text{ L} & 4 \text{ L} & 5 \text{ L} & 6 \text{ L} \\
\end{array}
\]
7.3.1 1 (a) 9 in. (b) 8 cm (c) 9 cm (d) 4 cm 2 (a) 4 years (b) $4000 3 (a) 2 years (b) 6% 4 (a) 2 (b) 4 5 (a) 32°F (b) –40°F 6 (a) 10 g (b) 8 cm³ 7 4 hr 48 min
8 46 mph 9 135 km 10 1 hr 25 min

7.4.1 1 (a) $x < 9$ (b) $x > -9$ (c) $x > 5$ (d) $x < 8$ (e) $x < 9$ (f) $x < -8$ (g) $x < 2$ (h) $x > 16$
2 (a) $x < 4$ (b) $x > 6$ (c) $x < 2$ (d) $x > -4/5$
3 (a) $x < 5/2$ (b) $x > 2$ (c) $x > 7$ (d) $x < 1$ (e) $x < 8$ (f) $x < 4$

7.5.1 1 $x = 5; y = 1$ 2 $x = 1; y = 1$ 3 $x = 1; y = 1$ 4 $x = 2; y = 1$
5 $x = 0; y = 2$ 6 $x = 2; y = 3$

7.5.2 1 $x = 3; y = 1$ 2 $x = 1; y = 4$ 3 $x = 3; y = 1$ 4 $x = -1; y = -1$
5 $x = 2; y = 2$ 6 $x = 2; y = 0$ 7 $x = 1; y = 1$ 8 $x = 1; y = 2$

Review Exercises
1 (a) Subtract; $x = 9$ (b) Add; $x = 15$ (c) Divide; $x = 4$
      (d) Multiply; $x = 36$
2 (a) $n = 4$ (b) $n = 21$ (c) $n = 13$ (d) $n = 2$ (e) $n = 6$
      (f) $n = 1$ (g) $n = 18$ (h) $n = 11$ (i) $n = -20$ (j) $q = 66$
      (k) $p = 36$ (l) $n = 12$ (m) $c = -10/3$
3 (a) $x = 3$ (b) $x = 20/3$ (c) $d = 6$ (d) $s = -1/5$ (e) $p = 2$
4 (a) $x = 66$ (b) $x = 180$ (c) $x = 1$ (d) $x = -0.3$
5 (a) $w = 5 \text{ cm}$ (b) $b = 10 \text{ in.}$ (c) $r = 10 \text{ cm}$ (d) $b = 12 \text{ cm}$
5 (e) $t = 4 \text{ years}$ (f) $r = 6\%$ (g) $C = -40\text{°C}$ (h) $F = -40\text{°F}$
6 (a) $x < 32$ (b) $x < 23$ (c) $x > 4$ (d) $x > 10$ (e) $x > -5$
5 (f) $x < -2$ (g) $x < -2$ (h) $x > 1/3$
7 (a) $x = 3; y = 2$ (b) $x = 1; y = 1$ (c) $x = 1; y = 1$
5 (d) $x = 2, y = 2$
8 (a) $x = 3; y = 1$ (b) $x = 1; y = 1$ (c) $x = 3; y = 2$
5 (d) $x = 4; y = -3$

Chapter 8
8.1.1 1

[Graph of stock prices over time]

[Graph of temperature changes over time]
ANSWERS TO EXERCISES

3

8.2.1

1

\[ N = 3M \]

\[
\begin{array}{c|cccccccc}
\text{Year} & 78 & 79 & 80 & 81 & 82 & 83 \\
\hline
\text{Income (\$ millions)} & 0.5 & 1.0 & 1.5 & 2.0 & 2.5 & 3.0 \\
\end{array}
\]

2

\[ N = M + 5 \]

3

\[ N = M - 1 \]

4

\[ N = M - 3 \]

5

\[ N = 2M + 3 \]
8.2.2 1

2
A (-6, 6)  H (-4, -2)
B (0, 7)  I (4, -2)
C (6, 7)  J (-6, -5)
D (-4, 3)  K (0, -5)
E (4, 4)  L (6, -4)
F (-6, 0)
G (6, 0)

8.3.1 1 \( y = 2x \)
2 \[ y = x + 4 \]

3 \[ y = -\frac{1}{3}x \]

4 \[ y = x - 4 \]
5 \ y = 2x + 1

6 \ y = 3x - 4

7 \ y = \frac{2}{3}x + 1
8  \( y = -x + 5 \)

9  \( y = -3x \)

10 \( y = -3x + 1 \)
11 \[ y = -x - 2 \]

12 \[ y = \frac{1}{2}x \]
13  \( y = -2x + 5 \)

14  \( y = -2x + 3 \)
15 \ y = -2x - 1

16 \ y = -\frac{1}{4}x - 2
17 \( y = 2 \)

18 \( x = -3 \)
19 \( y = -3 \)

20 \( x = 7 \)

8.4.1 1 (a) (0, 3) (b) (0, -4) (c) (0, 6) (d) (0, -2) (e) (0, 2) (f) (0, -3)

2 (a) 2 (b) -1 (c) 2 (d) -\( \frac{1}{3} \) (e) 0 (f) undefined
8.5.1  

1  \( m = 1; \ b = -2 \)

2  \( m = 5; \ b = 0 \)

3  \( m = 1/2; \ b = -5 \)
4 \quad m = -3; \quad b = -1

5 \quad m = 3; \quad b = 2

6 \quad m = -2/3; \quad b = 1
7 $m = 3/2; b = -2$

8 $m = 3/4; b = -6$

9 $m = -3/2; b = 3$
ANSWERS TO EXERCISES

10 \( m = \frac{2}{5}; b = -2 \)

Review Exercises

1

2 (a) \( N = 6 + M \)  
(b) \( N = 4M \)
(c) \( N = M - 3 \)

\[
\begin{array}{c|cccc}
N & 3 & 2 & 1 & 0 \\
\hline
M & 1 & 2 & 3 & 4
\end{array}
\]

(d) \( N = 1 + 2M \)

\[
\begin{array}{c|cccc}
N & 2 & 3 & 4 & 5 \\
\hline
M & 0 & 1 & 2 & 3
\end{array}
\]

3

4  A (0, 0), B (2, 2), C (-1, 6) D (0, 4), E (-7, 1), F (-2, -2),
G (0, -6), H (5, -1), I (8, 0)

5  (a) \( y = x + 2 \)
(b) \( y = x - 3 \)

(c) \( y = 3x \)
(d) \( y = 3 \)

\[ y = 3 \quad \text{when} \quad (0, 3), (1, 3), (2, 3) \]

(e) \( y = 2x + 1 \)

\[ y = 2x + 1 \quad \text{when} \quad (0, 1), (1, 3), (2, 5) \]
(f) \( x = 4 \)

(g) \( y = -x + 2 \)
(h) $y = 3x - 2$

6 (a) $m = \frac{3}{2}$
(b) \( m = 1 \)

(c) \( m = -\frac{7}{3} \)
ANSWERS TO EXERCISES

(d) $m = -\frac{1}{2}$

- Graph showing a line with points (0,2) and (4,0).

(e) $m = 0$

- Graph showing a horizontal line with points (4,2) and (8,2).
ANSWERS TO EXERCISES

(f) $m = -1$

7  (a) $m = 1; b = 2$
(b) \( m = 1; \ b = -5 \)

(c) \( m = 2; \ b = 0 \)
(d) \( m = -2; \ b = 4 \)

\[ \begin{array}{cccc}
\text{y} & 8 & 6 & 4 \\
\text{x} & 8 & 6 & 4 \\
\end{array} \]

\( (0,4) \)

\( (1,2) \)

(e) \( m = 1/2; \ b = 0 \)

\[ \begin{array}{cccc}
\text{y} & 8 & 6 & 4 \\
\text{x} & 8 & 6 & 4 \\
\end{array} \]

\( (0,0) \)

\( (2,1) \)
(f) $m = 0; b = -3$

(g) $m = -1; b = 6$
(h) \( m = 2; b = -2 \)

Chapter 9

9.1.1  1 \( 3 + 25 \)  2 \( 12 + 5 \)  3 \( 11 + 5 \)  4 \( 15 - 3 \)  5 \( 10 - 3 \)  6 \( 5(4 + 7) \)
  7 \( 2 - 9 \)  8 \( 5 + 6 \)  9 \( 12 - x \)  10 \( x - 5 \)  11 \( 5(6 + y) \)
  12 \((3p + 5q)4\)  13 \( 20 + x \)  14 \( 3t + x \)  15 \( 4x + (a + b) \)
  16 \( 5c(4x - 9y) \)

9.2.1  1 \( x + 4 = 10; x = 6 \)  2 \( x + 25 = 36; x = 11 \)  3 \( x - 3 = 20; x = 23 \)
  4 \( 7 - x = 11; x = -4 \)  5 \( 2x = 18; x = 9 \)
  6 \( 5x = 40; x = 8 \)  7 \( 2x + 3 = 31; x = 14 \)  8 \( 3x + 25 = 40; x = 5 \)
  9 \( 5x - 8 = 20; x = 5.6 \)  10 \( 8x - 9 = 7; x = 2 \)
  11 \( 2x + 13 = 25; x = 6 \)  12 \( 25 + 3x = 34; x = 3 \)
  13 \( 5x - 8 = 22; x = 6 \)  14 \( 6x + 7 = 37; x = 5 \)
  15 \( 2x = 5 + x; x = 5 \)  16 \( 7 + x = 2x; x = 7 \)

9.2.2  1 \( \frac{1}{2}x < 4; x < 20 \)  2 \((5 + x) \geq 26; x \geq 8 \)  3 \( \frac{3}{2}x \geq 10; x \geq 15 \)
  4 \( 15 - 2x \leq 41; x \geq -13 \)  5 \( 60 - 6x \leq 132; x \geq -12 \)
  6 \( \text{lunch} \leq 400; \text{breakfast} \leq 200; \text{dinner} \leq 600 \)
  7 \( \text{width} \leq 10.5 \text{ cm}, \text{length} \leq 31.5 \text{ cm} \)  8 \( \text{at least 91} \)

9.3.1  1 \( 7, 8, 9 \)  2 \( 17, 18, 19 \)  3 \( 13, 15, 17 \)  4 \( 20, 22, 24 \)  5 \( 16 \)
  6 \( 14, 18 \)  7 \( 10, 12 \)  8 \( 94 \)

9.3.2  1 \( 20\% \)  2 \( 16\% \)  3 \( 40\% \)  4 \( 30\% \)  5 \( 3.75\% \)  6 \( 14\% \)  7 \( 600 \)
  8 \( 80 \)  9 \( 16,000 \)  10 \( 45,000 \)  11 \( 400 \)  12 \( \$120 \)  13 \( \$25.95 \)
  14 \( \$4500 \)  15 \( \$126 \)  16 \( \$7000 \text{ at 10\%;} \$5000 \text{ at 14\%} \)
  17 \( \$800 \text{ at 6\%;} \$400 \text{ at 5\%} \)  18 \( \$401.50 \)

9.3.3  1 \( 45 \text{ ft} \)  2 \( 33 \text{ gal} \)  3 \( 24 \text{¢} \)  4 \( 7.5 \text{ gal} \)  5 \( \text{width 72 ft; length 108 ft} \)
  6 \( 24 \text{ in.}; 36 \text{ in.} \)  7 \( 10 \text{ acres corn}; 5 \text{ acres potatoes} \)  8 \( \$420 \)
ANSWERS TO EXERCISES

9.3.4  
1 $l = 18\text{ in.}$; $w = 9\text{ in.}$  
2 $w = 4\text{ ft}$; $l = 8\text{ ft}$  
3 $17\frac{1}{2}\text{ cm}$; $17\frac{1}{2}\text{ cm}$  
4 $w = 6\text{ units}$; $l = 20\text{ units}$  
5 $6\text{ ft}$; $6\text{ ft}$; $2\text{ ft}$  
6 $7\text{ m}$; $8\text{ m}$; $12\text{ m}$  
7 $55\text{ sq ft}$  
8 $14.9\text{ sq ft}$

9.3.5  
1 $3\text{ hr}$  
2 $46\text{ mph}$; $56\text{ mph}$  
3 $2\text{ mph}$  
4 (a) $1\text{ hr}$ (b) $4\text{ miles}$  
5 $192.5\text{ miles}$  
6 $17\text{ mph}$  
7 Jim $200\text{ mph}$; John $600\text{ mph}$  
8 $1.54\text{ mph}$

9.3.6  
1 Jane $9\text{ yr}$; Mary $24\text{ yr}$  
2 Liz $30\text{ yr}$; John $36\text{ yr}$  
3 Betsey $5\text{ yr}$; Father $36\text{ yr}$  
4 Casey $8\text{ yr}$; Aaron $4\text{ yr}$  
5 Fiori $11\text{ yr}$; Omn $8\text{ yr}$  
6 Deborah $34\text{ yr}$; Daughter $9\text{ yr}$  
7 Aaron $4\text{ yr}$; Cassandra $8\text{ yr}$  
8 David $35\text{ yr}$; Jonathan $42\text{ yr}$

9.3.7  
1 $18\text{ min}$  
2 $37\frac{1}{2}\text{ days}$  
3 Assad $18\text{ days}$ and Abduhl $36\text{ days}$  
4 $2\frac{1}{2}\text{ days}$  
5 $2\text{ hr 24 min}$  
6 $12\text{ hr}$  
7 $2:43\text{ P.M.}$

9.3.8  
1 $36\text{ dimes}$, $72\text{ pennies}$  
2 $14\text{ dimes}$, $33\text{ nickels}$  
3 $27\text{ qt} \times 28\text{¢}$, $18\text{ qts} \times 33\text{¢}$  
4 $1756\text{ adults}$, $744\text{ children}$  
5 $50\text{ @ 23¢}$, $100\text{ @ 37¢}$  
6 $66\text{ dimes}$, $80\text{ quarters}$  
7 $23.3\%$  
8 $20\text{ fl oz}$

9.4.1  
1 Lars $165\text{ cm}$; Hans $147\text{ cm}$  
2 wine $85\text{ gal}$; cider $35\text{ gal}$  
3 $25\text{ yr}$  
4 $84\text{ days}$  
5 In-state $6250\text{ stades}$; out-of-state $2500\text{ stades}$  
6 $23\text{ days}$  
7 Dave $30\text{ min}$; Deb $20\text{ min}$; Judy $60\text{ min}$

9.4.2  
1 $14\frac{1}{6}, 15\frac{1}{6}, 16\frac{1}{6}, 17\frac{1}{6}, 18\frac{1}{6}, 19\frac{1}{6}$  
2 $60\text{ years}$  
3 (a) $3600\text{ stades}$  
4 $14\frac{28}{97}\text{ stades}$  
5 $60\text{ drachmas}$  
6 $336\text{ walnuts}$  
7 $18\text{ mangos}$  
8 mother $218\frac{1}{7}$; daughter, $218\frac{3}{11}$

Review Exercises  
1 (a) $ab$ (b) $a + b$ (c) $a - b$ (d) $a/b$ (e) $a/b$ (f) $a - b$  
(g) $a - b$ (h) $a - b$  
2 (a) $x - 4 = 11$; $x = 15$ (b) $x/8 = 2$; $x = 16$  
(c) $8 + x = 2$; $x = 4$ (d) $2x - 5 = 11$; $x = 8$  
(e) $(x + 6) + 7 = 3$; $x = 15$ (f) $3x + 3 = 24$; $x = 7$  
3 $21, 22, 23$  
4 $11, 13, 15, 17$  
5 $22\%$  
6 $106.25$  
7 $36,000$  
8 $3\frac{1}{2}\text{ days}$  
9 $3000$  
10 $2\text{ yr}$  
11 $18\text{ yr}$  
12 Dick is $5\text{ yr}$ old; Tom is $10\text{ yr}$ old; Harry is $16\text{ yr}$ old  
13 Bob $17\text{ yr}$; Pete $7\text{ yr}$  
14 $1\text{ hr 20 min}$  
15 $9\text{ days}$  
16 $5\text{ quarters}$, $12\text{ dimes}$

Chapter 10  

10.1.1  
1 $4500$  
2 (a) $52$ (b) $40$ (c) $75$ (d) $80$ (e) $90$  
3 (a) $15$ (b) $24$ (c) $35$ (d) $128$ (e) $225.12$  
10.1.2  
1 Author $807.47$ Agent $121.12$  
2 $44\%$ $140$  
3 $49\%$  
4 $49\%$  
5 $18.8\%$