Exercises 1.1

Getting Ready
You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.
1. If a number satisfies an equation, it is called a _________ or a _________ of the equation.
2. If an equation is true for all values of its variable, it is called an _________.
3. A contradiction is an equation that is true for _________ values of its variable.
4. A _________ equation is true for some values of its variable and is not true for others.
5. An equation of the form $ax + b = 0$ is called a _________ equation.
6. If an equation contains rational expressions, it is called a _________ equation.
7. A conditional linear equation has _________ root.
8. The _________ of a fraction can never be 0.

Practice
Each quantity represents a real number. Find any restrictions on $x$.

9. $x + 3 = 1$ no restrictions
10. $\frac{1}{2}x - 7 = 14$ no restrictions
11. $\frac{1}{x} = 12$ $x \neq 0$
12. $\frac{3x}{x - 2} = 9x$ $x \neq 2$
13. $\frac{8}{x - 3} = \frac{5}{x + 2}$ $x \neq 3, x \neq 2$
14. $\frac{x}{x - 3} = \frac{4}{x + 4}$ $x \neq 3, x = -4$
15. $\frac{1}{x^2 - 16} = \frac{5x}{x - 3}$ $x \neq 3, x \neq 2$
16. $\frac{1}{x^2 - 3x - 4} = \frac{5}{x}$ $x \neq 0, 4, -1$

Solve each equation, if possible. Classify each one as an identity, a conditional equation, or a contradiction.

17. $2x + 5 = 15$ 5: conditional equation
18. $3x + 2 = x + 8$ 3: conditional equation
19. $2(n + 2) - 5 = 2n$ no solution; contradiction
20. $3(m + 2) = 2(m + 3) + m$ all real numbers; identity
21. $x + \frac{7}{2} = 7$ 7: conditional equation
22. $\frac{x}{2} - 7 = 14$ 42 conditional equation
23. $2(a + 1) = 3(a - 2) - a$ no solution; contradiction
24. $x^2 = (x + 4)(x - 4) + 16$ all real numbers; identity
25. $3(x - 3) = \frac{6x - 18}{2}$ all real numbers; identity
26. $x(x + 2) = (x + 1)^2$ no solution; contradiction
27. $\frac{3}{b - 3} = 1$ 6: conditional equation
28. $x^2 - 8x + 15 = (x - 3)(x + 5)$ 5: conditional equation
29. $2x^2 + 5x - 3 = (2x - 1)(x + 3)$ all real numbers; identity
30. $2x^2 + 5x - 3 = 2x\left(x + \frac{19}{2}\right)$ 14: conditional equation

Solve each equation. If an equation has no solution, so indicate.

31. $2x + 7 = 10 - x$ 1
32. $9a - 3 = 15 + 3a$ 3
33. $5(x - 2) = 2(x + 4)$ 6
34. $5(r - 4) = -5(r - 4)$ 4
35. $7(2x + 5) - 6(x + 8) = 7$ 5
36. $6(x - 5) - 4(x + 2) = -1$ 37
37. $\frac{5}{3}x - 8 = 7$ 9
38. $\frac{4}{3}y + 12 = -4$ 12
39. $\frac{x}{5} + 2 = 4$ 10
40. $\frac{3p}{7} - p = -4$ 7
41. $\frac{3x - 2}{3} = 2x + \frac{7}{3}$ 3
42. $\frac{7}{2}x + 5 = x + \frac{15}{2}$ 1
43. $\frac{3x + 1}{20} = \frac{1}{2}$ 3
44. $\frac{2x - 7}{6} + \frac{x}{2} = \frac{4x + 3}{6}$ 6
45. $\frac{3 + x}{3} + \frac{x + 7}{2} = 4x + 1$ 20
46. $\frac{2x + 1}{3} - \frac{3x}{2} = -\frac{3(4 + x)}{2}$ 22
47. $\frac{3}{2}(3x - 2) - 10x - 4 = 0$ 14
48. $\frac{a(a - 3) + 5}{7} = \frac{(a - 1)^2}{7}$ 4
49. $\frac{(y + 2)^2}{3} = y + 2 + \frac{y^2}{3}$ 2
50. $(r + 1)(r - 1) = (r + 2)(r - 3) + 4$ 1
51. $x(x + 2) = (x + 1)^2 - 1$ all real numbers
52. $(x - 2)(x - 3) = (x + 3)(x + 4)$ 1
53. $2(s + 2) + (s + 3)^2 = s(s + 5) + 2\left(\frac{17}{2} + s\right)$ 4
54. $\frac{3}{x} + \frac{1}{2} = \frac{4}{x}$ 2
55. $\frac{2}{x + 1} + \frac{1}{3} = \frac{1}{x + 1}$ no solution
56. $\frac{3}{x - 2} + \frac{1}{x} = \frac{3}{x - 2}$ no solution
Chapter 1  Equations and Inequalities

57. \[
9t + 6 = \frac{7}{t + 3}
\]
no solution

58. \[
x + \frac{2(-2x + 1)}{3x + 5} = \frac{3x^2}{3x + 5}
\]
\[-2\]

59. \[
\frac{2}{(a - 7)(a + 2)} = \frac{1}{(a + 3)(a + 2)}
\]
\[17\]

60. \[
\frac{2}{n - 2} + \frac{1}{n + 1} = \frac{1}{n^2 - n - 2}
\]
\[3\]

61. \[
\frac{2x + 3}{x^2 + 5x + 6} + \frac{3x - 2}{x^2 + x - 6} = \frac{5x - 2}{x^2 - 4}
\]
\[2\]

62. \[
\frac{3x}{x^2 + x} + \frac{2x}{x^2 + 5x} = \frac{1}{x^2 + 6x + 5}
\]
\[2\]

63. \[
\frac{3x}{x^3 + 8} + \frac{3}{x^2 - 4} = \frac{1}{(x - 2)(x^2 - 2x + 4)}
\]
\[3\]

64. \[
\frac{1}{5 + 42n + 16} = \frac{1}{5n + 2}
\]
\[2\]

65. \[
\frac{1}{11 - n} + \frac{1}{17n + 33} = \frac{1}{7n + 3}
\]
\[3\]

66. \[
\frac{a^2 - 13a - 48}{a^2 - 18a + 32} = \frac{1}{a^2 + a - 6}
\]
\[2\]

67. \[
\frac{5}{y + 4} + \frac{2}{y + 2} = \frac{6}{y^2 + 6y + 8}
\]
\[5\]

68. \[
\frac{2a - 6}{3 - 3a} = \frac{a^2 - 4a + 3}{a^2}
\]
\[7\]

69. \[
\frac{3y}{6 - 3y} + \frac{2y}{2y + 4} = \frac{1}{4 - y^2}
\]
no solution

70. \[
\frac{3 + 2a}{a^2 + 6 + 5a} - \frac{3 - 3a}{a^2 + 6 + a} = \frac{5a - 2}{a^2 - 4}
\]
\[5\]

71. \[
\frac{a}{a + 2} - 1 = \frac{3a + 2}{a^2 + 4a + 4}
\]
\[2\]

72. \[
\frac{x - 1}{x + 3} + \frac{x - 2}{x - 3} = \frac{1 - 2x}{3 - x}
\]
\[11\]

Solve each formula for the specified variable.

73. \[
k = 2.2p; p = \frac{k}{2.2}
\]

74. \[
ax + b = 0; x = -\frac{b}{a}
\]

75. \[
P = 2l + 2w; w
\]
\[p + 2l \div 2
\]

76. \[
V = \frac{1}{3} \pi r^2 h; h
\]
\[\frac{3V}{\pi r^2}
\]

77. \[
V = \frac{1}{3} \pi r^2 h; r^2
\]
\[\frac{7V}{\pi h}
\]

78. \[
z = \frac{x - \mu}{\sigma}; \mu
\]
\[\mu = x - z\sigma
\]

79. \[
P = L + \frac{si}{f}; f
\]
\[s = \frac{P - L}{i}
\]

80. \[
P = L + \frac{si}{f}; f
\]
\[s = \frac{P - L}{i}
\]

81. \[
F = \frac{mMg}{r^2}; m_m = \frac{r^2F}{Mg}
\]
\[\frac{1}{f} = \frac{1}{p} + \frac{1}{q}; f = \frac{pq}{q + p}
\]

82. \[
\frac{x}{a} + \frac{y}{b} = 1; y
\]
\[y = \frac{b}{a} \left(1 - \frac{y}{a}\right)
\]

83. \[
\frac{a}{b} - \frac{b}{a} = 1; a
\]
\[a = \frac{b}{a + y}
\]

84. \[
\frac{a}{b} + \frac{b}{a} = 1; a
\]
\[a = \frac{b}{a + y}
\]

85. \[
\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}; r
\]
\[r = \frac{r_1 r_2}{r_1 + r_2}
\]

86. \[
\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}; r
\]
\[r = \frac{r_1 r_2}{r_1 + r_2}
\]

87. \[
l = a + (n - 1)d; d
\]
\[n = \frac{l - a + d}{d}
\]

88. \[
l = a + (n - 1)d; d
\]
\[d = \frac{l - a}{n - 1} = \frac{a}{1 - n}
\]

89. \[
a = (n - 2)\frac{180}{n}; n
\]
\[n = \frac{360}{180 - a}
\]

90. \[
S = a - br; a
\]
\[a = S - Sr + br
\]

91. \[
R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}; r_1
\]
\[r_1 = \frac{r_1 r_2}{r_1 + r_2}
\]

92. \[
R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}; r_3
\]
\[r_3 = \frac{r_1 r_2}{r_1 + r_2}
\]

Discovery and Writing

93. Explain why a conditional linear equation always has exactly one root.

94. Define an extraneous solution and explain how such a solution occurs.

Review

Simplify each expression. Use absolute value symbols when necessary.

95. \[
(25x^3)^{1/2} 5|x|, 25|x|^3
\]

96. \[
\left(\frac{25p^2}{16q^2}\right)^{1/2} 5/p, \frac{25p^2}{16q^2}
\]

97. \[
\left(\frac{125x^3}{8y^6}\right)^{-2/3} 4x^4, \frac{25x^8}{25y^4}
\]

98. \[
\left(-\frac{27y^3}{1000x^6}\right)^{1/3} -3y, 3y
\]

99. \[
\sqrt[3]{25y^2}; 5y
\]

100. \[
-\sqrt[3]{125y^9}; -5y
\]

101. \[
\sqrt[3]{\frac{a^4b^12}{x^5y^5}} = \sqrt[3]{\frac{a^4b^{12}}{x^5y^5}}
\]

102. \[
\sqrt[3]{\frac{a^10b^5}{x^5y^5}} = \sqrt[3]{\frac{a^10b^5}{x^5y^5}}
\]
8. Test scores Courtney took four tests in science class. On each successive test, her score improved by 3 points. If her mean score was 69.5%, what did she score on the first test? 65%

9. Teacher certification On the Illinois certification test for teachers specializing in learning disabilities, a teacher earned the scores shown in the accompanying table. What was the teacher’s score in program development? 74

| Human development with special needs | 82 |
| Assessment                        | 90 |
| Program development and instruction | ? |
| Professional knowledge and legal issues | 78 |
| AVERAGE SCORE                     | 86 |

10. Golfing Par on a golf course is 72. If a golfer shot rounds of 76, 68, and 70 in a tournament, what will she need to shoot on the final round to average par? 74

11. Replacing locks A locksmith at Pop-A-Lock charges $40 plus $28 for each lock installed. How many locks can be replaced for $236? 7

12. Delivering ads A University of Florida student earns $20 per day delivering advertising brochures door-to-door, plus 75¢ for each person he interviews. How many people did he interview on a day when he earned $56? 48

13. Electronic LED billboard An electronic LED billboard in Times Square is 26 feet taller than it is wide. If its perimeter is 92 feet, find the dimensions of the billboard. 10 ft by 36 ft

14. Hockey rink A National Hockey League rink is 115 feet longer than it is wide. If the perimeter of the rink is 570 feet, find the dimensions of the rink? 200 ft by 85 ft

15. Width of a picture frame The picture frame with the dimensions shown in the illustration was built with 14 feet of framing material. Find x its width. 2.5 ft

16. Fencing a garden If a gardener fences in the total rectangular area shown in the illustration instead of just the square area, he will need twice as much fencing to enclose the garden. How much fencing will he need? 96 ft

17. Wading pool dimensions The area of the triangular swimming pool shown in the illustration is doubled by adding a rectangular wading pool. Find the dimensions of the wading pool. (Hint: The area of a triangle = \( \frac{1}{2}bh \), and the area of a rectangle = \( lh \).) 20 ft by 8 ft

18. House construction A builder wants to install a triangular window with the angles shown in the illustration. What angles will he have to cut to make the window fit? (Hint: The sum of the angles in a triangle equals 180°.) 70°, 70°, 40°

19. Length of a living room If a carpenter adds a porch with dimensions shown in the illustration to the living room, the living area will be increased by 50%. Find the length of the living room. 20 ft
20. **Depth of water in a trough** The trough in the illustration has a cross-sectional area of 54 square inches. Find the depth, $d$, of the trough. (*Hint: Area of a trapezoid = \( \frac{1}{2}h(b_1 + b_2) \)) \( d = 5.4 \text{ in.} \)

![](image)

21. **Investment problem** An executive invests $22,000, some at 7% and some at 6% annual interest. If he receives an annual return of $1,420, how much is invested at each rate? $10,000 at 7\%, \$12,000 at 6\%$

22. **Financial planning** After inheriting some money, a woman wants to invest enough to have an annual income of $5,000. If she can invest $20,000 at 9\% annual interest, how much more will she have to invest at 7\% to achieve her goal? (See the table.) $43.714.29$

<table>
<thead>
<tr>
<th>Type</th>
<th>Rate</th>
<th>Amount</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>9% investment</td>
<td>0.09</td>
<td>20,000</td>
<td>$0.09(20,000)$</td>
</tr>
<tr>
<td>7% investment</td>
<td>0.07</td>
<td>$x$</td>
<td>$0.07x$</td>
</tr>
</tbody>
</table>

23. **Investment problem** Equal amounts are invested at 6\%, 7\%, and 8\% annual interest. If the three investments yield a total of $2,037 annual interest, find the total investment. $29,100$

24. **Investment problem** A woman invests $37,000, part at 8\% and the rest at 9\% annual interest. If the 9\% investment provides $452.50 more income than the 8\% investment, how much is invested at each rate? $17,500 at 8\%, \$19,500 at 9\%$

25. **Ticket sales** A full-price ticket for a college basketball game costs $2.50, and a student ticket costs $1.75. If 585 tickets were sold, and the total receipts were $1,217.25, how many tickets were student tickets? 327

26. **Ticket sales** Of the 800 tickets sold to a movie, 480 were full-price tickets costing $7 each. If the gate receipts were $4,960, what did a student ticket cost? $5

27. **Discounts** After being discounted 20\%, a weather radio sells for $63.96. Find the original price. $79.95$

28. **Markups** A merchant increases the wholesale cost of a Maytag washing machine by 30\% to determine the selling price. If the washer sells for $588.90, find the wholesale cost. $453$

29. **Break-point analysis** A machine to mill a brass plate has a setup cost of $600 and a unit cost of $3 for each plate manufactured. A bigger machine has a setup cost of $800 but a unit cost of only $2 for each plate manufactured. Find the break point. 200 units

30. **Break-point analysis** A machine to manufacture fasteners has a setup cost of $1,200 and a unit cost of $0.005 for each fastener manufactured. A newer machine has a setup cost of $1,500 but a unit cost of only $0.0015 for each fastener manufactured. Find the break point. $95.714.29$

31. **Computer sales** A computer store has fixed costs of $8,925 per month and a unit cost of $850 for every computer it sells. If the store can sell all the computers it can get for $1,275 each, how many must be sold for the store to break even? (Hint: The break-even point occurs when costs equal income.) 21

32. **Restaurant management** A restaurant has fixed costs of $137.50 per day and an average unit cost of $4.75 for each meal served. If a typical meal costs $6, how many customers must eat at the restaurant each day for the owner to break even? 110

33. **Roofing houses** Kyle estimates that it will take him 7 days to roof his house. A professional roofer estimates that it will take him 4 days to roof the same house. How long will it take if they work together? $2\frac{6}{11} \text{ days}$

34. **Sealing asphalt** One crew can seal a parking lot in 8 hours and another in 10 hours. How long will it take to seal the parking lot if the two crews work together? $4\frac{1}{9} \text{ hr}$

35. **Mowing lawns** Julie can mow a lawn with a lawn tractor in 2 hours, and her husband can mow the same lawn with a push mower in 4 hours. How long will it take to mow the lawn if they work together? $1\frac{1}{3} \text{ hr}$

36. **Filling swimming pools** A garden hose can fill a swimming pool in 3 days, and a larger hose can fill the pool in 2 days. How long will it take to fill the pool if both hoses are used? $1\frac{1}{2} \text{ days}$

37. **Filling swimming pools** An empty swimming pool can be filled in 10 hours. When full, the pool can be drained in 19 hours. How long will it take to fill the empty pool if the drain is left open? $21\frac{1}{9} \text{ hr}$

38. **Preparing seafood** Kevin stuffs shrimp in his job as a seafood chef. He can stuff 1,000 shrimp in 6 hours. When his sister helps him, they can stuff 1,000 shrimp in 4 hours. If Kevin gets sick, how long will it take his sister to stuff 500 shrimp? 6 hours

39. **Diluting solutions** How much water should be added to 20 ounces of a 15\% solution of alcohol to dilute it to a 10\% solution? $10 \text{ oz}$
40. **Increasing concentrations** The beaker shown below contains a 2% saltwater solution.
   a. How much water must be boiled away to increase the concentration of the salt solution from 2% to 3%? 100 ml
   b. Where on the beaker would the new water level be? 200-ml mark

![Beaker diagram]

41. **Winterizing cars** A car radiator has a 6-liter capacity. If the liquid in the radiator is 40% antifreeze, how much liquid must be replaced with pure antifreeze to bring the mixture up to a 50% solution? 1 liter

42. **Mixing milk** If a bottle holding 3 liters of milk contains 3% butterfat, how much skimmed milk must be added to dilute the milk to 2% butterfat? 2/3 liter

43. **Preparing solutions** A nurse has 1 liter of a solution that is 20% alcohol. How much pure alcohol must she add to bring the solution up to a 25% concentration? 2/15 liter

44. **Diluting solutions** If there are 400 cubic centimeters of a chemical in 1 liter of solution, how many cubic centimeters of water must be added to dilute it to a 25% solution? (Hint: 1,000 cc = 1 liter.) 600 cm³

45. **Cleaning swimming pools** A swimming pool contains 15,000 gallons of water. How many gallons of chlorine must be added to “shock the pool” and bring the water to a 30% solution? About 4.5 gal

46. **Mixing fuels** An automobile engine can run on a mixture of gasoline and a substitute fuel. If gas costs $3.50 per gallon and the substitute fuel costs $2 per gallon, what percent of a mixture must be substitute fuel to bring the cost down to $2.75 per gallon? 54.5%

47. **Evaporation** How many liters of water must evaporate to turn 12 liters of a 24% salt solution into a 36% solution? 4 liters

48. **Increasing concentrations** A beaker contains 320 ml of a 5% saltwater solution. How much water should be boiled away to increase the concentration to 6%? 53 1/3 ml

49. **Lowering fat** How many pounds of extra-lean hamburger that is 7% fat must be mixed with 30 pounds of hamburger that is 15% fat to obtain a mixture that is 10% fat? 50 lb

50. **Dairy foods** How many gallons of cream that is 22% butterfat must be mixed with milk that is 2% butterfat to get 20 gallons of milk containing 4% butterfat? 2 gal

51. **Mixing solutions** How many gallons of a 5% alcohol solution must be mixed with 90 gallons of a 1% solution to obtain a 2% solution? 30 gal

52. **Preparing medicines** A doctor prescribes an ointment that is 2% hydrocortisone. A pharmacist has 1% and 5% concentrations in stock. How much of each should the pharmacist use to make a 1-ounce tube? 3/4 oz of 1% + 1/4 oz of 5%

53. **Driving rates** John drove to Daytona Beach, Florida, in 5 hours. When he returned, there was less traffic, and the trip took only 3 hours. If John averaged 26 mph faster on the return trip, how fast did he drive each way? 39 mph going; 65 mph returning

54. **Distance problem** Allison drove home at 60 mph, but her brother Austin, who left at the same time, could drive at only 48 mph. When Allison arrived, Austin still had 60 miles to go. How far did Allison drive? 300 mi

55. **Distance problem** Two cars leave Hinds Community College traveling in opposite directions. One car travels at 60 mph and the other at 64 mph. In how many hours will they be 310 miles apart? 3 1/8 hr

56. **Bank robbery** Some bank robbers leave town, speeding at 70 mph. Ten minutes later, the police give chase, traveling at 78 mph. How long, after the robbery, will it take the police to overtake the robbers? 1 1/8 hr

57. **Jogging problem** Two Michigan State University cross-country runners are 440 yards apart and are running toward each other, one at 8 mph and the other at 10 mph. In how many seconds will they meet? 50 sec

58. **Driving rates** One morning, Justin drove 5 hours before stopping to eat lunch at Pizza Hut. After lunch, he increased his speed by 10 mph. If he completed a 430-mile trip in 8 hours of driving time, how fast did he drive in the morning? 50 mph

59. **Boating problem** A Johnson motorboat goes 5 miles upstream in the same time it requires to go 7 miles downstream. If the river flows at 2 mph, find the speed of the boat in still water. 12 mph

60. **Wind velocity** A plane can fly 340 mph in still air. If it can fly 200 miles downwind in the same amount of time it can fly 140 miles upwind, find the velocity of the wind. 60 mph