Solids of Revolution Practice Problems

Find the volume of the solid obtained by rotating the region bounded by the given curves about the given axis. Use the method of slicing (disks/washers). Sketch the region and a typical disk or washer.

1. \( y = x^2 - 2x, y = 8; \) about the line \( y = 8 \)
2. \( y = x^2, x = 1, y = 0; \) about the line \( y = 1 \)
3. \( y = x^2, x^2 + y^2 = 2; \) about the line \( y = -2 \) [Set this up only.]
4. \( y = x^2, x = 1, y = 0; \) about the line \( x = 1 \)
5. \( y = \frac{1}{x}, x = -2, x = -1, \) about the line \( x = -1 \)
6. \( y = x^2, y = x; \) about the line \( x = -2 \)

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the axis indicated. Sketch the region and a typical shell.

7. \( y = x^2, x = 1, y = 0; \) about the line \( y = 1 \)
8. \( y = x^2, x = 1, y = 0; \) about the line \( y = -1 \)
9. \( y = x^2, x^2 + y^2 = 2; \) about the line \( y = -2 \) [Set this up only.]
10. \( y = x^2, x = 1, y = 0; \) about the line \( x = 1 \)
11. \( y = \frac{1}{x}, x = -2, x = -1, \) about the line \( x = -1 \)
12. \( y = x^2, y = x; \) about the line \( x = -2 \)

If you have some strange desire to try and work out problems \#3 and \#9, you may make use of the fact that:

\[
\int \sqrt{2-x^2} \, dx = \frac{x}{2} \sqrt{2-x^2} + \arcsin\left(\frac{x}{\sqrt{2}}\right) + C
\]

Answers:

<table>
<thead>
<tr>
<th></th>
<th>1. ( \frac{1296\pi}{5} )</th>
<th>2. ( \frac{7\pi}{15} )</th>
<th>3. ( 2\pi^2 + \frac{64\pi}{15} )</th>
<th>4. ( \frac{\pi}{6} )</th>
<th>5. ( 2\pi - \pi\ln4 )</th>
<th>6. ( \frac{5\pi}{6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>( \frac{7\pi}{15} )</td>
<td>8. ( \frac{13\pi}{15} )</td>
<td>9. ( 2\pi^2 + \frac{64\pi}{15} )</td>
<td>10. ( \frac{\pi}{6} )</td>
<td>11. ( 2\pi - \pi\ln4 )</td>
<td>12. ( \frac{5\pi}{6} )</td>
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