Section 6.1 - Discrete Random Variables
Probability Distribution

Definitions

a) Random variable - is a variable whose values are determined by chance.
b) Discrete Probability distribution consists of the values a random variable can assume and the corresponding probabilities of the values.

TWO Requirements for a Probability distribution

a) All probabilities must be between 0 and 1
b) The sum of the probabilities must add up to 1.

- Discrete RANDOM VARIABLE - The number of values that X can assume is FINITE or COUNTABLE
- Continuous RANDOM VARIABLE - The number of values that X can assume is INFINITE. (IE MEASURE)

Example #1
Construct a probability distribution for drawing a card from a deck of 40 cards consisting of 10 cards numbered 1, 10 cards numbered 2, 15 cards numbered 3, and 5 cards numbered 4.

Example #2 (Number 26 in your book, section 6.1)
The following data represents (in millions) the enrollment levels in grades 9th to 12th in the US in 2000.

a) Construct a discrete probability distribution for the random variable X
b) Draw the probability histogram

<table>
<thead>
<tr>
<th>X (Grade Level)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>3958</td>
</tr>
<tr>
<td>10</td>
<td>3487</td>
</tr>
<tr>
<td>11</td>
<td>3080</td>
</tr>
<tr>
<td>12</td>
<td>2799</td>
</tr>
</tbody>
</table>
Mean Variance, Standard Deviation, and Expectation

c) Mean for a probability distribution is found by using the following formula:
\[
\mu = \sum X \cdot P(X)
\]

d) Variance for a probability distribution:
\[
\sigma^2 = \sum (X^2 \cdot P(X)) - \mu^2
\]

e) Standard Deviation = \[\sigma = \sqrt{\sigma^2}\]

f) Expectation: \[E(X) = \mu\]
(The expected value of a random variable, written \[E(X)\] is the same as the mean \[\mu\] of a random variable)

Example #3
Suppose that the number of car accidents per day in Torrance, CA has been recorded for the past year. The table below is a probability distribution table representing the data collected.

<table>
<thead>
<tr>
<th>Number of Accidents per day</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.12</td>
<td>.32</td>
<td>.25</td>
<td>.13</td>
<td>??</td>
</tr>
</tbody>
</table>

a) What is the probability that on a given day, the number of accidents will be 4?

b) On any given day, how many accidents do you expect to have?

c) What is the standard deviation?
Example #5 (Number 22 in your book)
A Wendy's manager performed a study to determine a probability distribution for the number of people waiting in line \( X \) during lunch. The results were as follows:

\[ \text{USE CALCULATOR!!} \]

<table>
<thead>
<tr>
<th>( X )</th>
<th>( P(X) )</th>
<th>( X )</th>
<th>( P(X) )</th>
<th>( X )</th>
<th>( P(X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.011</td>
<td>5</td>
<td>0.172</td>
<td>10</td>
<td>0.019</td>
</tr>
<tr>
<td>1</td>
<td>0.035</td>
<td>6</td>
<td>0.132</td>
<td>11</td>
<td>0.002</td>
</tr>
<tr>
<td>2</td>
<td>0.089</td>
<td>7</td>
<td>0.098</td>
<td>12</td>
<td>0.006</td>
</tr>
<tr>
<td>3</td>
<td>0.150</td>
<td>8</td>
<td>0.063</td>
<td>13</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.186</td>
<td>9</td>
<td>0.035</td>
<td>14</td>
<td>0.001</td>
</tr>
</tbody>
</table>

a) Verify that this is a discrete probability distribution.

b) Compute and interpret the mean of the random variable \( X \)

c) Compute the variance of the random variable \( X \)

d) Compute the standard deviation of the random variable \( X \)

e) What is the probability that there are eight people waiting in line for lunch?

f) What is the probability that there are 10 or more people waiting in line for lunch? Would this be unusual?
Example #6
A Roulette wheel has 38 compartment numbered 0, 00, 1, 2, 3, 4, 5, ...., 36. Half of 1 through 36 are red and the other half are colored black. The remaining two compartments numbered 0, and 00 are colored green. Suppose $1 is placed on a red compartment. When a ball is spun and lands in a red compartment, the player gets his $1 back plus $1. Otherwise, loses $1. What should the player expect to win in one game? In 100 games?

a) What is the random variable? Explain why?

b) What are the values that the random variable assumes? Explain why?

c) Find the corresponding probabilities?

d) Compute the expected value.

Example #7 (Number 36 in your book)
Some standardized tests, such as the SAT test, incorporate a penalty for wrong answers. For example, a multiple-choice question with five possible answers will have 1 point awarded for a correct answer and \( \frac{1}{4} \) deducted point for an incorrect answer. Questions left blank are worth 0 points. Find the expected number of points received for a multiple-choice question with five possible answers when a student just guesses.
Section 6.2 - Binomial Distribution

Definitions:

a) **Binomial experiment** - A binomial experiment is a probability experiment that satisfies the following four requirements
   1. There is a fixed number of trials
   2. There are only two outcomes or they can be reduced to TWO.
   3. The outcomes of each trial must be independent of each other.
   4. The probability of a success must remain the same for each trial.

b) Binomial Distribution is a distribution with the outcomes and its corresponding probabilities

c) **Binomial Probability Formula** - In a binomial experiment, the probability of exactly \( x \) successes in \( n \) trials is

\[
P(x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{n}{x} p^x q^{n-x}
\]

Where
- \( x = \) the number of successes
- \( p = \) the probability of success
- \( q = \) the probability of failure
- \( n = \) the number of trials

d) **Mean for a binomial distribution** = \( \mu = np \)

e) **Variance for a binomial distribution** = \( \sigma^2 = npq \)

f) **Standard Deviation for a binomial distribution** = \( \sigma = \sqrt{npq} \)

Example #1
A college claims that 70% of students receive financial aid. Suppose that 4 students at the college are randomly selected. We are interested in the number of students in the sample who receive financial aid.

a) Explain why this is a binomial experiment.
b) What is the random variable?

c) What are the possible values for X.

<table>
<thead>
<tr>
<th>X</th>
<th>Using complete sentences, write a description for each “x-value”</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>


d) Find the probability that there are exactly 2 students in the sample that receive financial aid.

- Using the formula
- Using calculator


e) Find the probability that there are exactly 3 students in the sample that receive financial aid

- Using the formula
- Using calculator
f) Construct a Probability Distribution.

<table>
<thead>
<tr>
<th>X</th>
<th>Work to find probabilities</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

g) Find the probability that there are at most 2 students in the sample that receive financial aid.

h) Find the probability that there are at least 2 students in the sample that receive financial aid.

i) Find the mean and the standard deviation and write a sentence explaining what the mean represents.
Example #2 – Using Calculator!!
According to American Airlines, its flight 215 from Orlando to Los Angeles is on time 90% of the time. Suppose 15 flights are randomly selected.

a) Explain why this is a binomial experiment

b) Find the probability that exactly 14 flights are on time.

c) Find the probability that at least 14 flights are on time.

d) Find the probability that fewer than 14 flights are on time.

e) Find the probability that between 12 and 14 flights, inclusive, are on time.
Example #3
A pediatrician is interested in how many of his patients with flu developed pneumonia as a complication. The doctor has 200 patients with flu this winter. It is estimated that 2.6% of children with flu developed pneumonia as a complication. We are interested in the number of his patients with flu who develop pneumonia.

a) Describe the random variable and the possible values it can assume.

b) Find the probability that 3 patients with flu develop pneumonia.

c) Find the probability that at least 10 patients with flu will develop pneumonia.

d) Find the probability that less than half of his patients will develop pneumonia.