**Math Review:**

**Exponents:**

```
     b
   ↙
a  ↘ EXPO
   ↙NENT
  ↙ BASE
```

**For example:**

```
 3
2  THIS MEANS 2 MULTIPLIED BY
   ITSELF 3 TIMES
```

\[ 2^3 = 2 \cdot 2 \cdot 2 = 8 \]

**If the base is negative:**

\[ (-2)^3 \]

\[ (-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8 \]

**Remember:**

NEG • NEG = POS
NEG • POS = NEG
IF THE EXPONENT IS NEGATIVE:

\[ \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8} \]

THE RULE IS:

\[ (\text{base})^{-\text{exponent}} = \frac{1}{\text{base}^{\text{exponent}}} \]

EXAMPLES OF EXPONENTS:

\[
\begin{align*}
 x^1 &= x \\
 3^1 &= 3 \\
 x^0 &= 1 \\
 3^0 &= 1 \\
 x^{-1} &= \frac{1}{x} \\
 m/s^{-1} &= \frac{m}{s} \\
 m^{-2} &= \frac{1}{m^2} \\
 \frac{g}{\text{mol}} &= g \cdot \text{mol}^{-1} \\
 \frac{M}{s} &= M \cdot s^{-1} \\
 \frac{g}{\text{cm}^3} &= g \cdot \text{cm}^{-3}
\end{align*}
\]

ADDING EXPONENTS:

\[ 3^2 + 3^2 = 9 + 9 = 18 \]

\[ x^2 + x^3 = \text{CANT ADD!} \]

\[ x^2 + x^2 = 2x^2 \]

Remember: PLEASE → PARENTHESIS
ORDER OF OPERATIONS
EXCUSE → EXPONENTS
MY → MULTIPLICATION
DEAR → DIVISION
AUNT → ADDITION
SALLY → SUBTRACTION

EXONENTS FIRSTS
THEN ADD
**Subtracting Exponents:**

**SAME AS ADDITION**

\[ 3^2 - 3^3 = 9 - 27 = -18 \]

\[ \text{Exponents First} \rightarrow \text{Then Subtract} \]

**Multiplying Exponents:**

\[ x^2 \cdot x^3 = (x \cdot x) \cdot (x \cdot x \cdot x) = x^5 \]

\[ \text{Shortcut:} \quad x^{2+3} \]

**Numbers in Front of the Base:**

\[ 3x^2 \cdot 3x^3 = 9x^{2+3} = 9x^5 \]

\[ \text{Multiply like normal} \rightarrow \text{Add Exponents} \]

**Example:**

\[ (3.10 \text{ cm}) \cdot (10.22 \text{ cm}^2) = 31.7 \text{ cm}^3 \]
**Dividing Exponents:**

\[ \frac{x^2}{x^3} = \frac{x^2}{x^3} = \frac{(x \cdot x)}{(x \cdot x \cdot x)} = \frac{1}{x} = x^{-1} \]

\[ \rightarrow \text{Shortcut: } x^{2-3} \]

**Example:**

\[ x = \frac{4.03 \text{ cm}^2}{2.96 \text{ cm}} = 1.36 \text{ cm} \]

**Raising a Power to a Power:**

\[ (x^{-2})^3 = (x^{-2})(x^{-2})(x^{-2}) = (\frac{1}{x^2})(\frac{1}{x^2})(\frac{1}{x^2}) = \frac{1}{x^6} = x^{-6} \]

\[ \rightarrow \text{Shortcut: } x^{-2}(3) \]
Logarithms: Exponents & logarithms are related:

Exponents:

\[ a^b = c \]

WE HAVE "a" AND "b" "c"

WE WANT "c"

Logarithms:

\[ \log_{10} b = c \]

WE HAVE "a"

"a" AND "c"

WE WANT "b"

For \(10^b = 100\), we would write:

\[ \log_{10} 100 = b = 2 \]

"Log-base ten"

Common logarithm:

Log-base ten is referred to as the "common log", when we write \(\log\) by itself, we assume log-base ten:

\[ \log_b = \log_{10} \]

On a calculator, "\(\log\)" means common log-
Some noteworthy logarithms:

\[ \log_q (\text{NEGATIVE NUMBER}) = \text{DOES NOT EXIST} \]
\[ \text{(NOTHING CAN MAKE } 10^x = \text{NEGATIVE \#)} \]

\[ \log_1 0 = \text{DOES NOT EXIST} \] (NOTHING CAN MAKE \( 10^x = 0 \))
\[ \log_1 0.001 = -3 \]
\[ \log_1 0.01 = -2 \]
\[ \log_1 0.1 = -1 \]
\[ \log_1 1 = 0 \]
\[ \log_1 10 = 1 \]
\[ \log_1 100 = 2 \]
\[ \log_1 1000 = 3 \]

Natural logarithm:

The natural logarithm describes the following exponential expression:

\[ e^x = y \Rightarrow \log_e y = x \]

We abbreviate: \( \log_e \) as \( \ln \) on calculator

"e" is a ubiquitous mathematical constant that equals approximately 2.718

\[ e = \lim_{x \to 0} (1+x)^{\frac{1}{x}} \]
Noteeworthy Natural Logarithms:

\[ \ln \text{(negative #)} = \text{does not exist} \]

(Nothing can make \( e^x = \text{a negative #} \))

\[ \ln 0 = \text{does not exist} \] (Nothing can make \( e^x = 0 \))

\[ \ln 0.001 = -6.908 \]
\[ \ln 0.01 = -4.605 \]
\[ \ln 0.1 = -2.303 \]
\[ \ln 1 = 0 \]
\[ \ln 2.718 = 1 \]
\[ \ln 10 = 2.303 \]
\[ \ln 100 = 4.605 \]
\[ \ln 1000 = 6.908 \]

Relationship of \( \log \) to \( \ln \):

\[ \ln x \approx 2.303 \log x \]

\[ 0.434 \ln x \approx \log x \]

For example:

\[ \ln 4 = 1.386 \]
\[ 2.303 \log 4 = 1.387 \]

\[ 0.434 \ln 2 = 0.301 \]
\[ \log 2 = 0.301 \]
**DISTRIBUTION & FACTORING:**

**DISTRIBUTION:** \( A(B + C) \rightarrow AB + AC \)

**FACTORING:** \( AB + AC \rightarrow A(B + C) \)

**EXAMPLES:**

\[ x(3x + 4x^2) = 3x^2 + 4x^3 \]

\[ 3x^2y^2 + x^2 = x^2(3y^2 + 1) \]

\[ (x + 4)(x + 2) = x^2 + 2x + 4x + 8 \]

\[ = x^2 + 6x + 8 \quad \text{(A QUADRATIC)} \]

**FACTORIZING A QUADRATIC:**

\[ x^2 + 6x + 8 = (x \quad)(x \quad) \]

\[ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \]

From adding "Outer & Inner" \quad From multiplying "Lasts" \quad Factors of 8 \quad 2 \cdot 4 = 8 \quad 2 + 4 = 6

\[ \downarrow \quad \downarrow \]

\[ = (x + 2)(x + 4) \]
SOLVING EQUATIONS FOR A VARIABLE:

\[ x = y - 3 \]

**Solve for** \( y \)

\[ x + 3 = y \]

**Add 3 to both sides**

\[ PV = nRT \]

**Solve for** \( T \)

\[ \frac{PV}{nR} = T \]

**Divide both sides by** \( nR \)

\[ \frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} \]

**Solve for** \( T_2 \)

\[ \frac{P_2 V_2 n_1 T_1}{P_1 V_1 n_2} \]

**Two ways....**

1) **Multiply both sides by** \( T_2 \)

- Multiply both sides by \( n_1 T_1 \)
- Divide both sides by \( P_1 V_1 \)

\[ T_2 = \frac{P_2 V_2 n_1 T_1}{P_1 V_1 n_2} \]

2) **Cross multiply**:

\[ \frac{P_1 V_1}{n_1 T_1} \times \frac{P_2 V_2}{n_2 T_2} \Rightarrow P_1 V_1 n_2 T_2 = P_2 V_2 n_1 T_1 \]

**Divide both sides by** \( P_1 V_1 n_2 \)

\[ T_2 = \frac{P_2 V_2 n_1 T_1}{P_1 V_1 n_2} \]
ALGEBRA & LOGARITHMS

WE USE THE FOLLOWING RELATIONSHIPS:

\[ \log_a b = \log_a c + \log_a d \]

\[ \log_a b = \log_a c - \log_a d \]

\[ \log_{10} x = x \]

\[ e^x = x \]

EXAMPLES:

TO SOLVE FOR \( x \):

\[ \log_{10} x = 4 \]

"ANTI-LOGARITHM"

RAISE BOTH SIDES TO A POWER OF 10

\[ 10^{\log_{10} x} = 10^4 \]

\[ x = 10^4 \]

\[ \ln x = 4 \]

RAISE BOTH SIDES TO A POWER OF "e"

\[ e^\ln x = e^4 \]

\[ x = e^4 \]

\[ -\log_{10} [H^+] = 9.86 \]

RAISE BOTH SIDES TO A POWER OF 10

\[ 10^{-\log_{10} [H^+]} = 10^{-9.86} \]

REARRANGE

\[ [H^+] = 10^{-9.86} \]

\[ \ln A = 24.62 \]

RAISE BOTH SIDES TO A POWER OF "e"

\[ e^{\ln A} = e^{24.62} \]

↑ MOST CALCULATORS HAVE AN \( e^x \) BUTTON

\[ A = e^{24.62} \]
IF TWO QUANTITIES HAVE A LINEAR RELATIONSHIP YOU CAN PLOT ONE VERSUS THE OTHER ON A GRAPH & A LINE CAN BE FIT THROUGH THE POINTS.

EQUATION FOR A LINE:

\[ y = mx + b \]

For any two points \((x_1, y_1)\) & \((x_2, y_2)\):

\[ \text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \]

\(y\)-INTERCEPT \((x=0, \text{ so } y=b)\)

\(x\)-INTERCEPT \(y=0, \text{ so } 0 = mx + b\)

\[ x = -\frac{b}{m} \]
Let's use density of some substance as an example.

\[ \text{Density} = \frac{\text{Mass}}{\text{Volume}} \]

Mass & Volume are linearly related.

\[ y = mx + b \]

Units are (g)

\[ \begin{align*}
  \text{Units} &= \text{(g)} \\
  \text{Volume} &= \text{(mL)} \\
  \text{Mass} &= \text{(g)} \\
  \text{Units} \text{ at } x &= \text{(g)} \\
  \text{Units} \text{ at } y &= \text{(g)}
\end{align*} \]

\[ \text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - 2\text{g}}{4\text{mL} - 2\text{mL}} = \frac{2\text{g}}{2\text{mL}} = \frac{\text{g}}{\text{mL}} \]

The slope in this case is equal to density (1 g/mL).

Note: Logarithms can not have units. However, sometimes it is okay to add units after the calculation.

Slope always has units of \( \frac{y}{x} \).
If you have a density of 2.50 g·cm⁻³ with a y-intercept of 0, what volume in cm³ will 17.4 g have?

\[ y = mx + b \]

\[ y = 2.50 \text{ g·cm}^{-3} \quad b = 0 \]

If \( y = 17.4 \text{ g} \)

\[ 17.4 \text{ g} = (2.50 \text{ g·cm}^{-3})x \]

\[ x = \frac{17.4 \text{ g}}{2.50 \text{ g·cm}^{-3}} \]

\[ x = 6.96 \text{ cm}^3 \]
The equation for a line is a linear "First Degree" equation (no exponents).

The quadratic equation is a parabolic "Second Degree" equation (highest power is 2)

\[
y = ax^2 + bx + c
\]
\[
0 = ax^2 + bx + c
\]
\[
\Rightarrow\ y = 0
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

"Quadratic Equation"

Let's solve one for fun!

\[
49x^2 - 102x + 49 = 0
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{(102) \pm \sqrt{(-102)^2 - 4(49)(49)}}{2(49)}
\]

\[
x = 1.33
\]

\[
x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{(102) - \sqrt{(-102)^2 - 4(49)(49)}}{2(49)}
\]

\[
x = 0.75
\]

Note:
The Sharp EL-506W calculator has a quadratic function. You enter a, b & c and it solves for x. It only costs about $14.00 (F2k's)

Usually in chemistry only one value of x will make sense.